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The online appendix to “Financial frictions in Latvia”, published in Empirical Economics

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1 Computational appendix

Table 1: Estimated foreign SVAR parameters

	Description	Distr.	Prior Mean	Prior st.d.	Posterior Mean	Posterior st.d.	HPD int. 10%	HPD int. 90%
ρ_{μ_z}	Persistence, unit-root tech.	β	0.50	0.075	0.590	0.063	0.487	0.696
a_{11}	Foreign SVAR parameter	N	0.90	0.05	0.913	0.034	0.852	0.977
a_{22}	Foreign SVAR parameter	N	0.50	0.05	0.521	0.055	0.438	0.605
a_{33}	Foreign SVAR parameter	N	0.90	0.05	0.954	0.023	0.919	0.989
a_{12}	Foreign SVAR parameter	N	-0.10	0.10	-0.165	0.091	-0.314	-0.016
a_{13}	Foreign SVAR parameter	N	-0.10	0.10	-0.045	0.054	-0.124	0.037
a_{21}	Foreign SVAR parameter	N	0.10	0.10	0.181	0.043	0.097	0.260
a_{23}	Foreign SVAR parameter	N	-0.10	0.10	-0.090	0.055	-0.183	-0.008
a_{24}	Foreign SVAR parameter	N	0.05	0.10	0.078	0.041	0.009	0.146
a_{31}	Foreign SVAR parameter	N	0.05	0.10	0.080	0.029	0.032	0.131
a_{32}	Foreign SVAR parameter	N	-0.10	0.10	-0.095	0.058	-0.198	0.002
a_{34}	Foreign SVAR parameter	N	0.10	0.10	0.108	0.026	0.068	0.149
c_{21}	Foreign SVAR parameter	N	0.05	0.05	0.021	0.040	-0.048	0.088
c_{31}	Foreign SVAR parameter	N	0.10	0.05	0.145	0.031	0.094	0.196
c_{32}	Foreign SVAR parameter	N	0.40	0.05	0.374	0.053	0.286	0.459
c_{24}	Foreign SVAR parameter	N	0.05	0.05	0.065	0.046	-0.003	0.135
c_{34}	Foreign SVAR parameter	N	0.05	0.05	0.048	0.034	-0.002	0.101
	Standard deviations of shocks							
$100\sigma_{\mu_z}$	Unit root technology	Inv-Γ	0.25	inf	0.328	0.052	0.248	0.406
$100\sigma_{y^*}$	Foreign GDP	Inv-Γ	0.50	inf	0.317	0.055	0.219	0.415
$1000\sigma_{\pi^*}$	Foreign inflation	Inv-Γ	0.50	inf	0.593	0.118	0.394	0.805
$100\sigma_{R^*}$	Foreign interest rate	Inv-Γ	0.075	inf	0.067	0.008	0.054	0.079

Note: Based on a single Metropolis-Hastings chain with 100 000 draws after a burn-in period of 900 000 draws.

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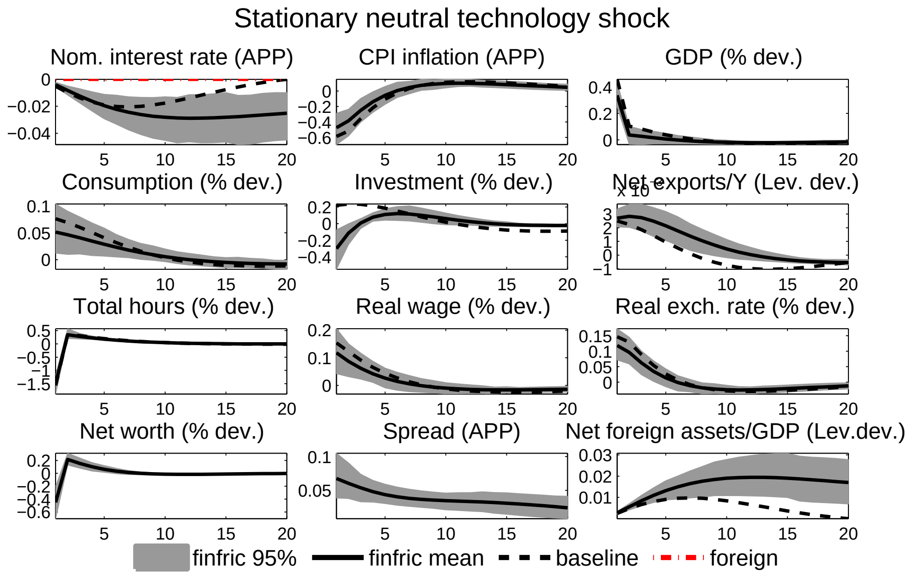

Fig. 1: Impulse responses to the stationary neutral technology shock, ϵ_t.

Note: The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev. dev.).

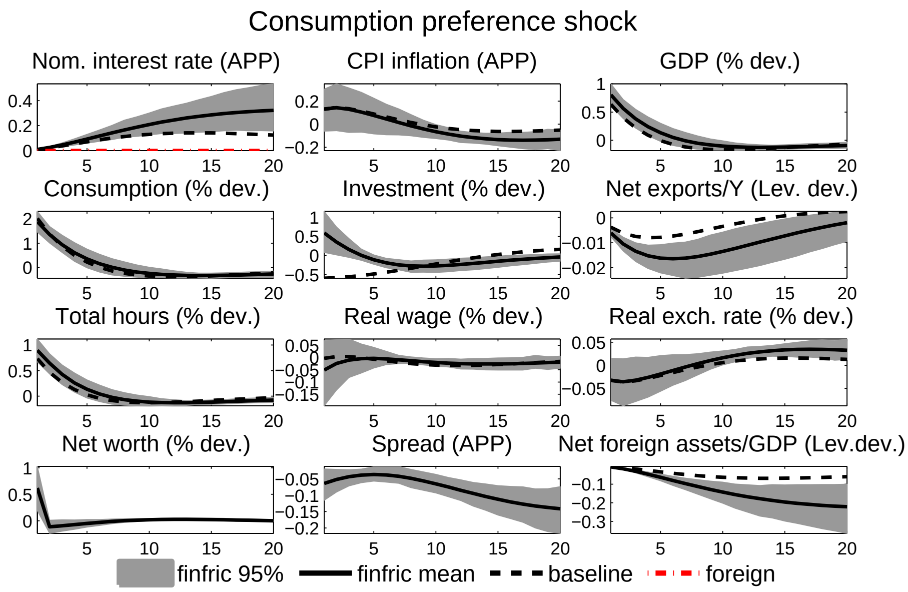

Fig. 2: Impulse responses to the consumption preference shock, ζ_t^c.

Note: The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev. dev.).

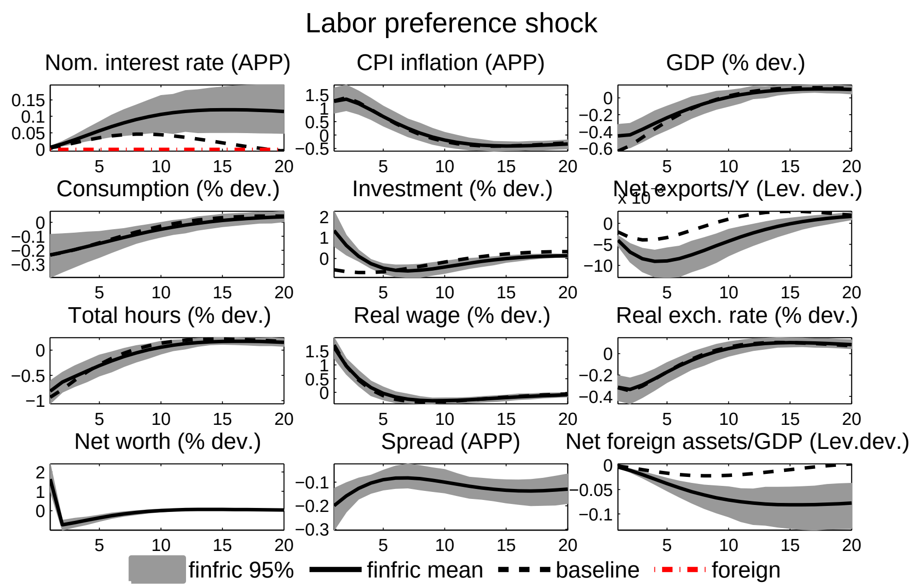

Fig. 3: Impulse responses to the labor preference shock, ζ_t^h.

Note: The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev. dev.).

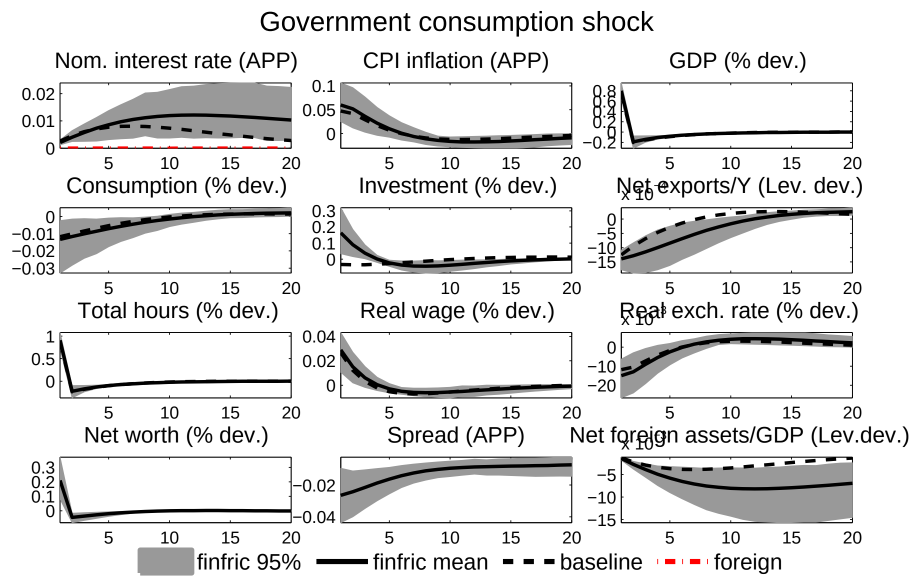

Fig. 4: Impulse responses to the government consumption shock, g_t.

Note: The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev. dev.). In this model, the government consumption crowds out the private consumption. Total consumption falls due to the worsening of the net foreign assets position and a subsequent increase in the risk premium to the nominal interest rate that makes saving activity more appealing.

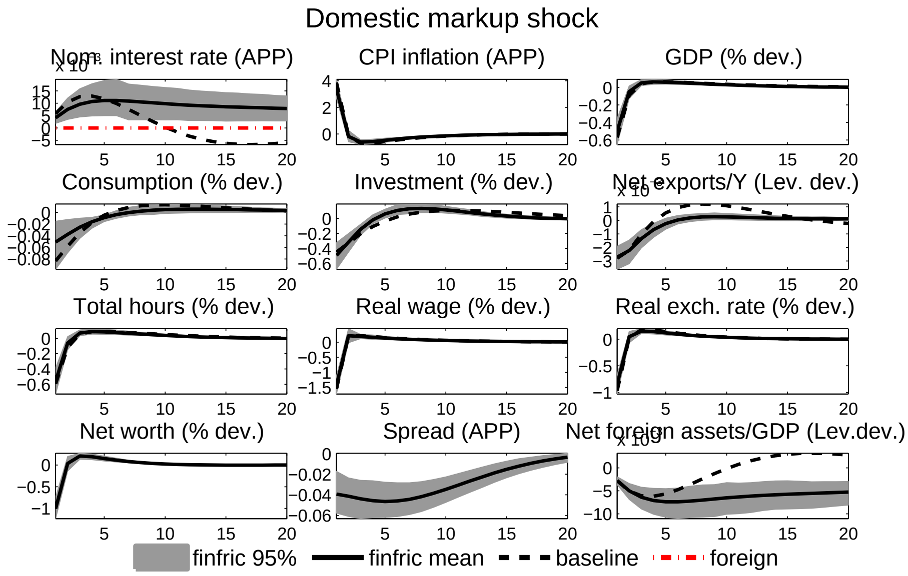

Fig. 5: Impulse responses to the domestic markup shock, τ_t^d.

Note: The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev. dev.).

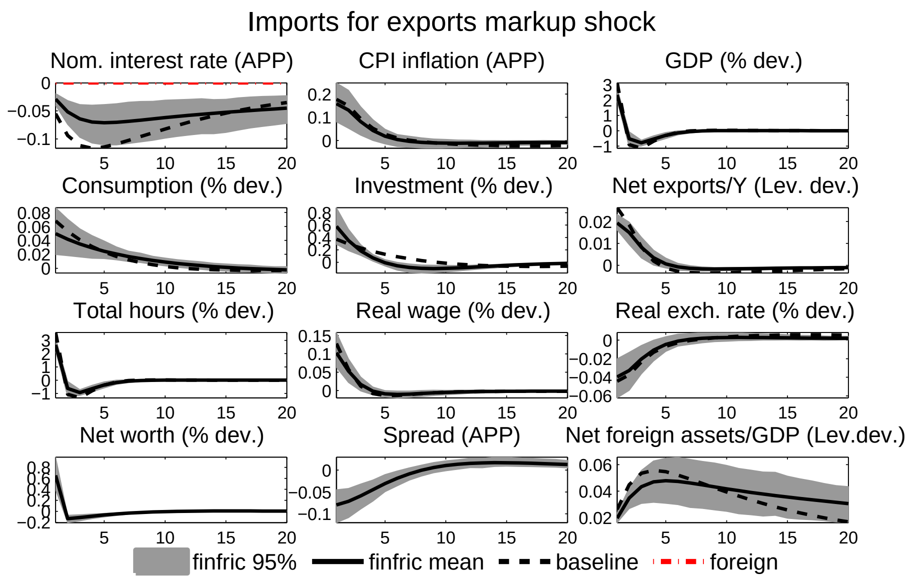

Fig. 6: Impulse responses to the imports for exports markup shock, τ_t^{mx}.

Note: The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev. dev.).

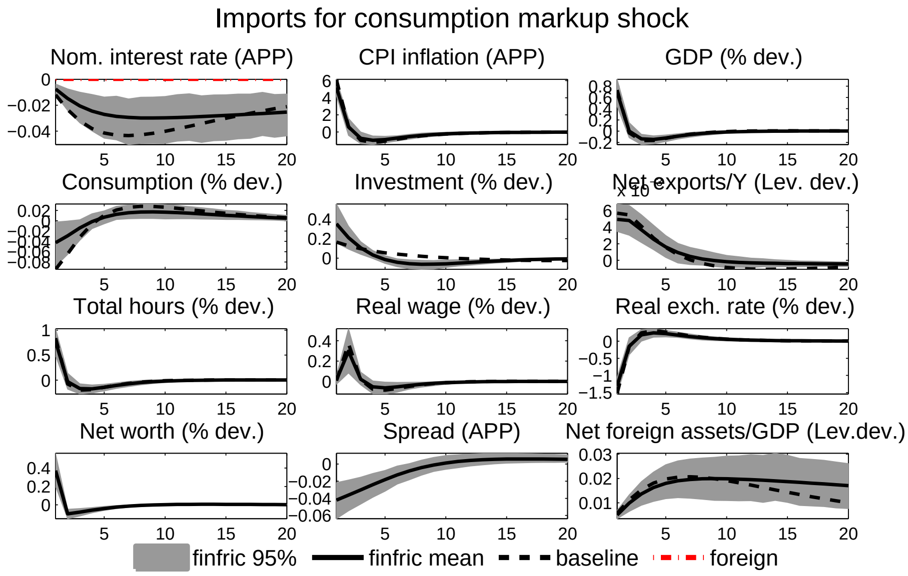

Fig. 7: Impulse responses to the imports for consumption markup shock, τ_t^{mc}.

Note: The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev. dev.).

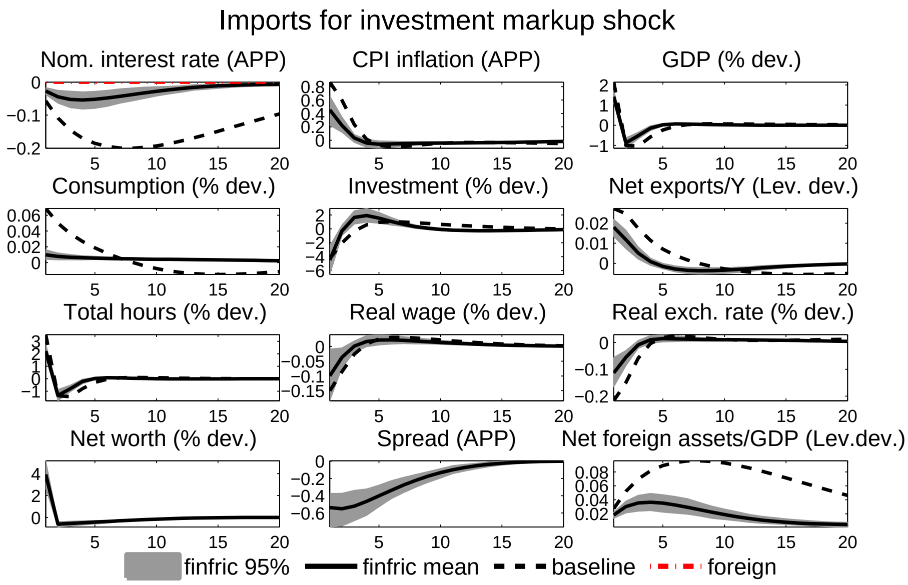

Fig. 8: Impulse responses to the imports for investment markup shock, τ_t^{mi}.

Note: The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev. dev.).

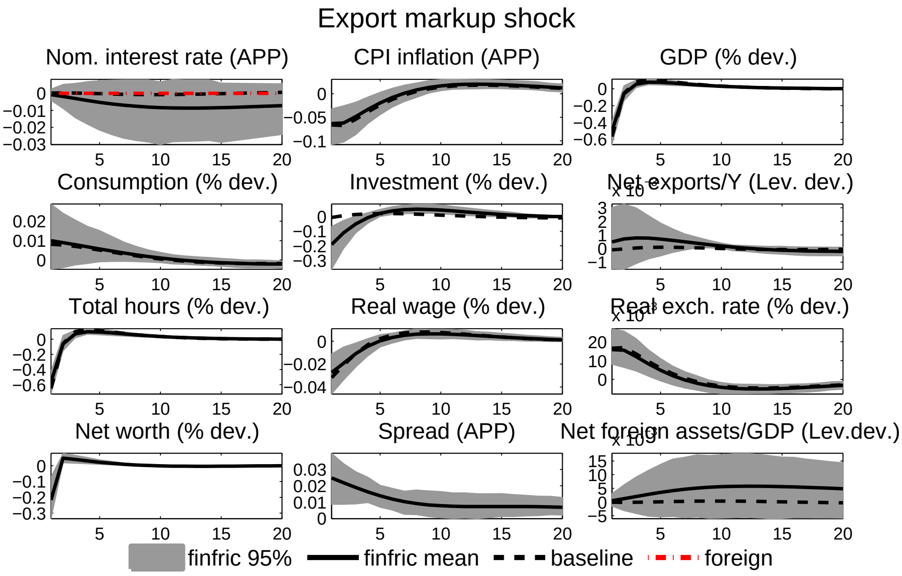

Fig. 9: Impulse responses to the export markup shock, τ_t^x.

Note: The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev. dev.).

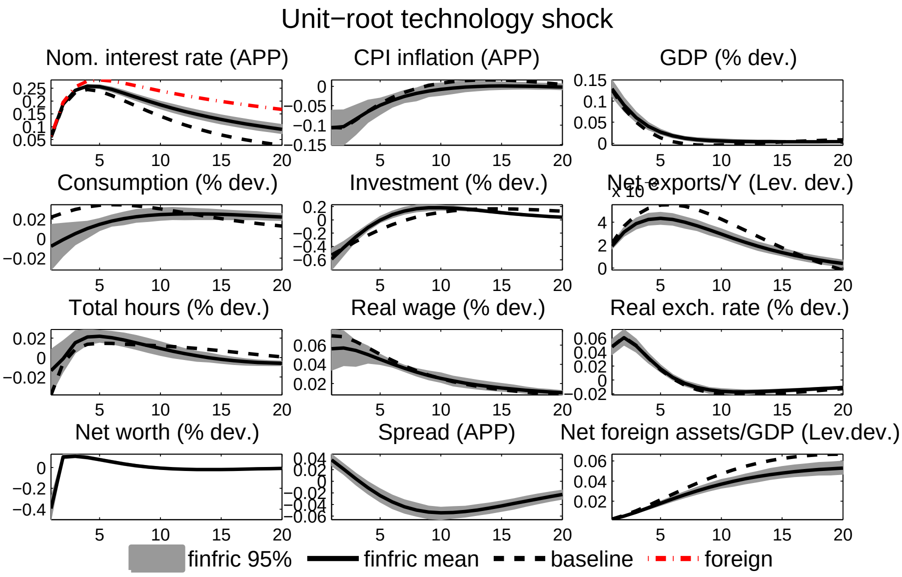

Fig. 10: Impulse responses to the unit-root technology shock, $\mu_{z,t}$.

Note: The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev. dev.).

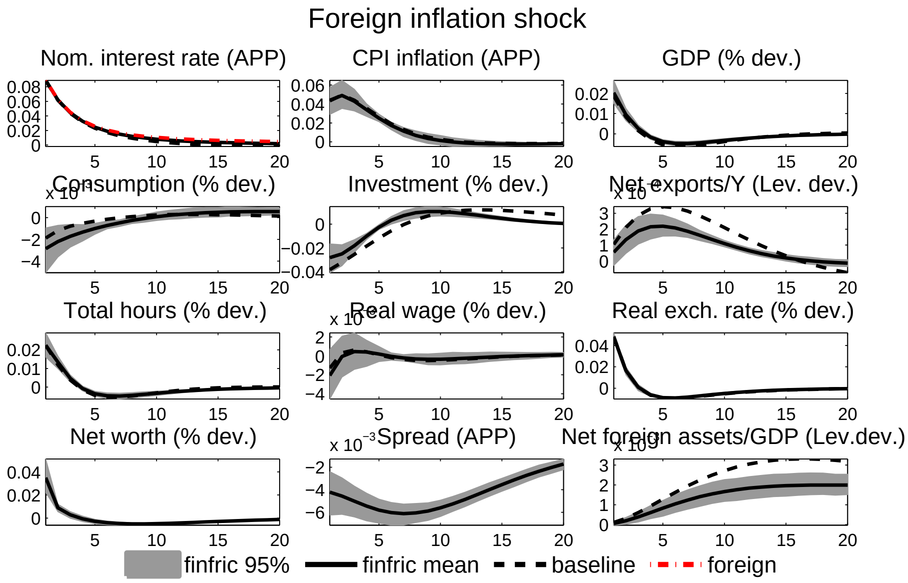

Fig. 11: Impulse responses to the foreign inflation shock, $\epsilon_{\pi^*,t}$.

Note: The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev. dev.). A temporary positive shock to foreign inflation causes the cost of imported consumption and investment to rise. As a result, consumption and investment decrease, imports decrease, and GDP goes up. The effects are small in magnitude, though.

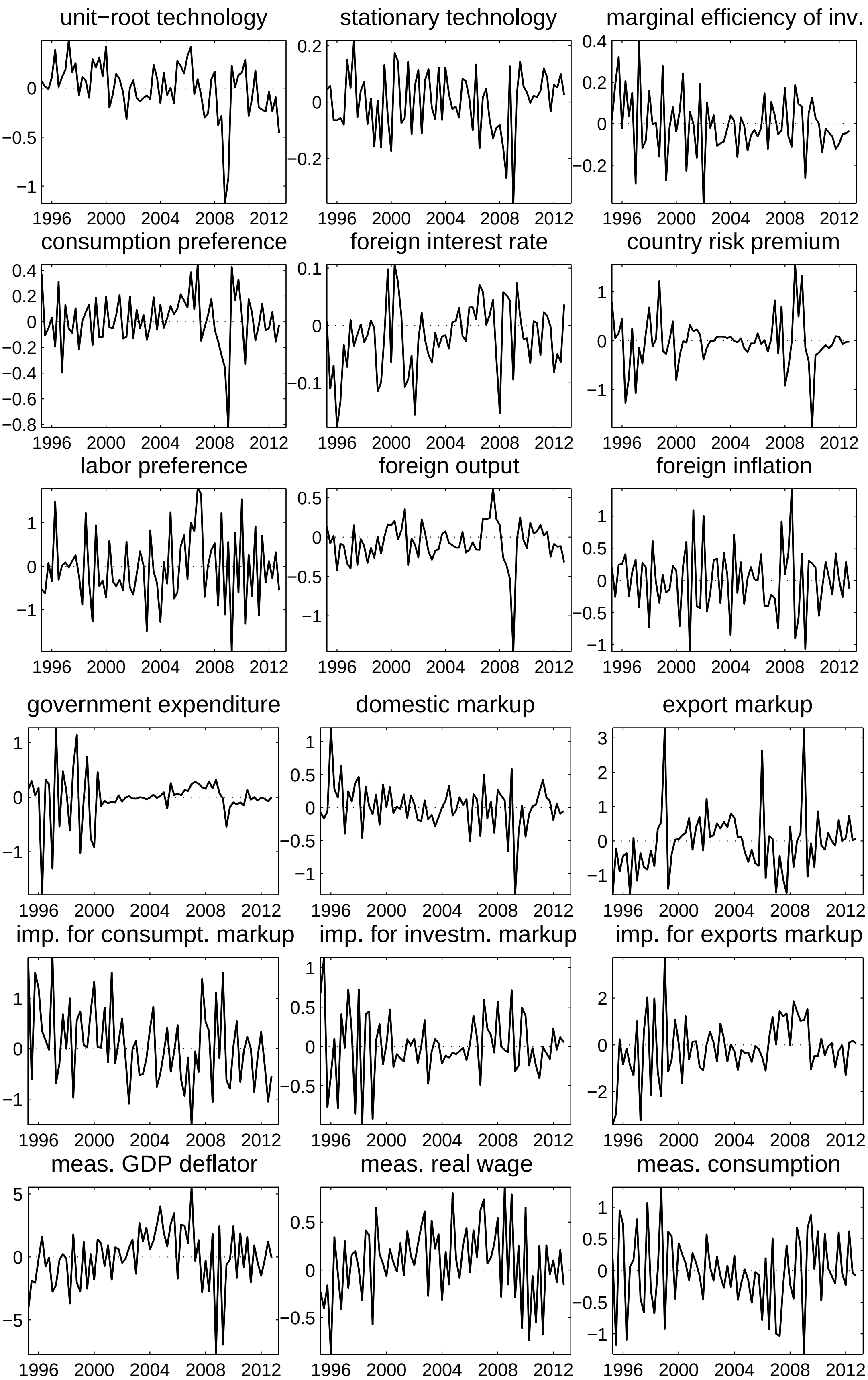

Fig. 12: Smoothed shock processes and measurement errors of the financial frictions model.

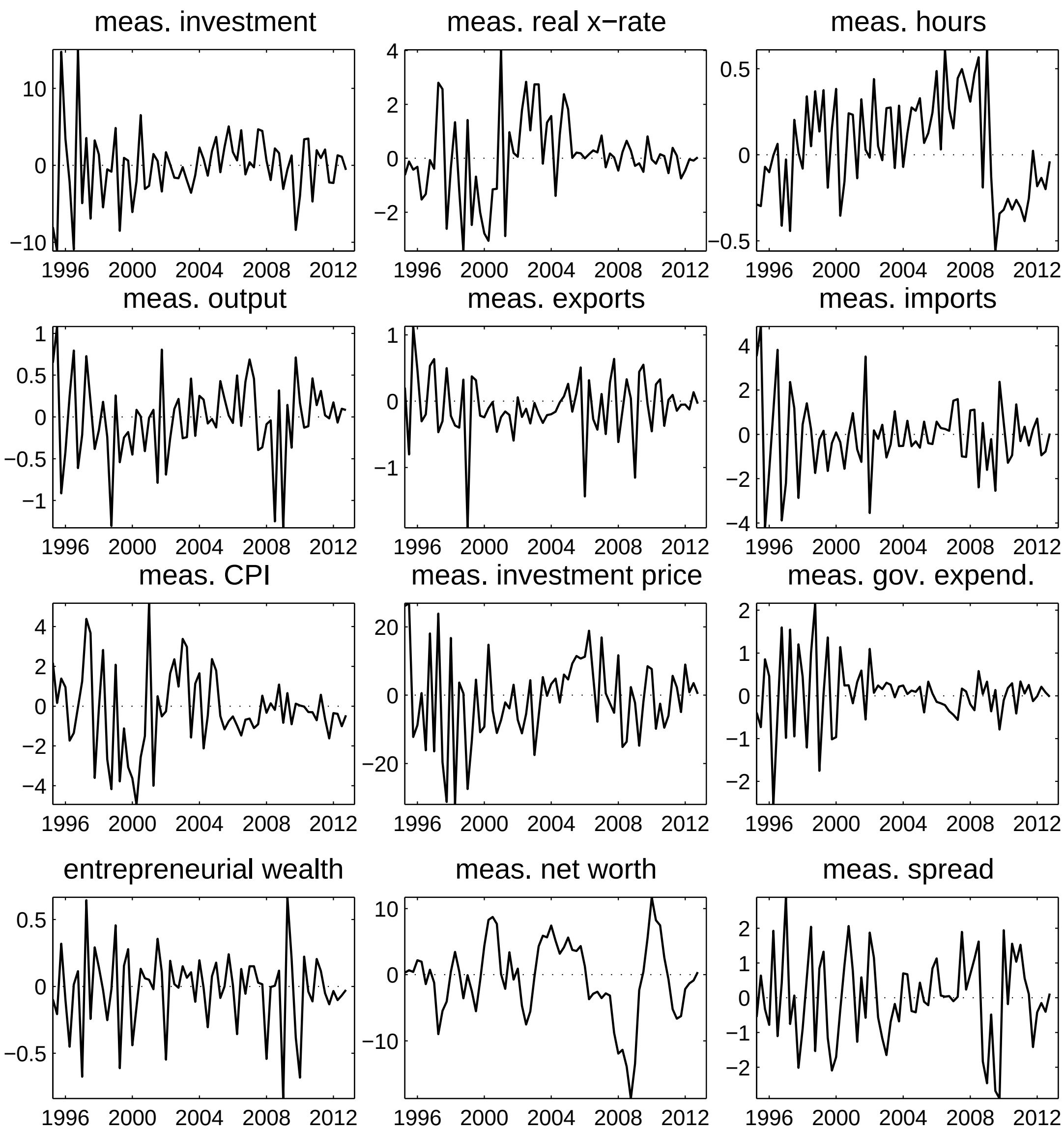

Fig. 13: Smoothed shock processes and measurement errors of the financial frictions model (continued).

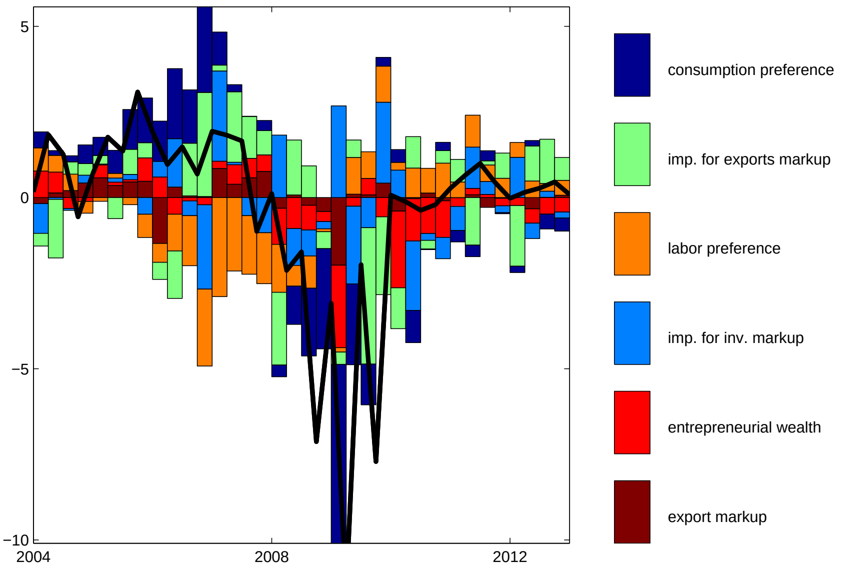

Fig. 14: Decomposition of GDP (quarterly growth rates), 2004Q1-2012Q4.

Note: Financial frictions model. Only the six shocks with the greatest influence shown.

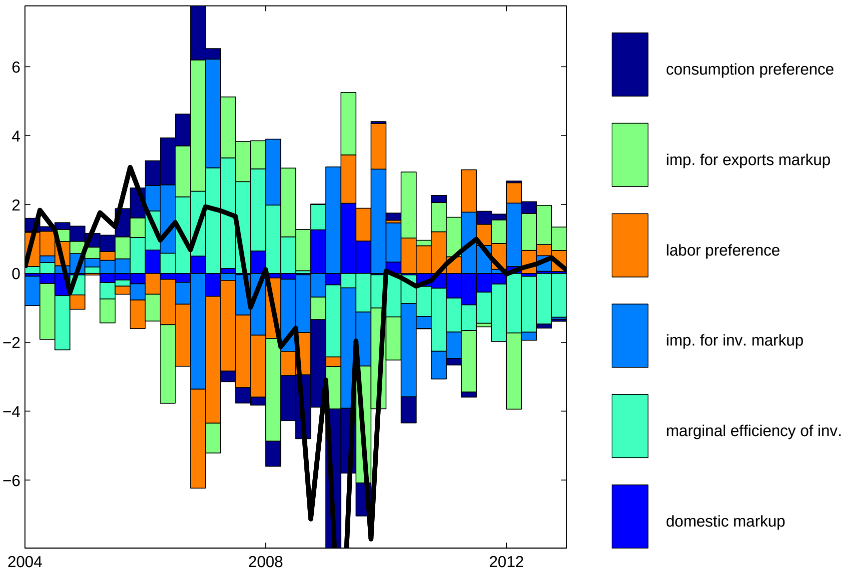

Fig. 15: Decomposition of GDP (quarterly growth rates), 2004Q1-2012Q4, Baseline model.

Only the six shocks with the greatest influence shown.

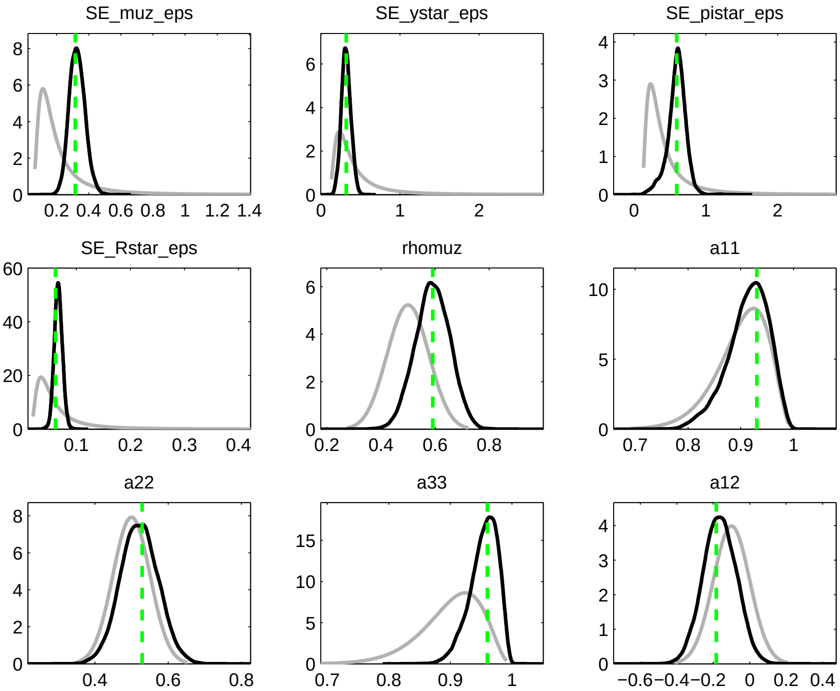

Fig. 16: SVAR priors and posteriors.

Note: Prior distribution in gray, simulated distribution in black, and the computed posterior mode in dashed green.

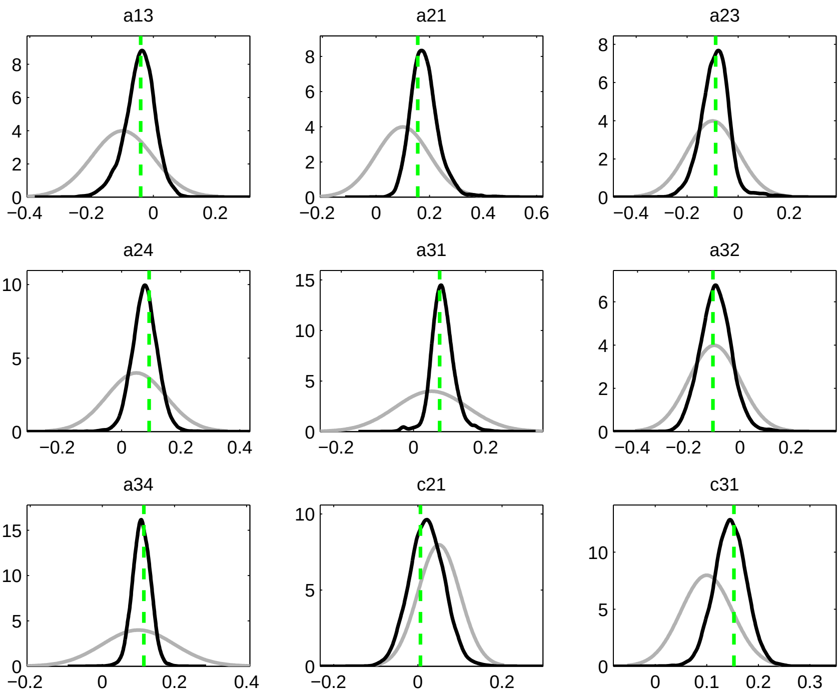

Fig. 17: SVAR priors and posteriors (continued).

Note: Prior distribution in gray, simulated distribution in black, and the computed posterior mode in dashed green.

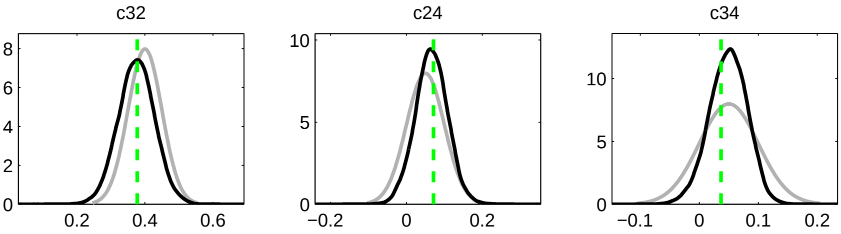

Fig. 18: SVAR priors and posteriors (continued).

Note: Prior distribution in gray, simulated distribution in black, and the computed posterior mode in dashed green.

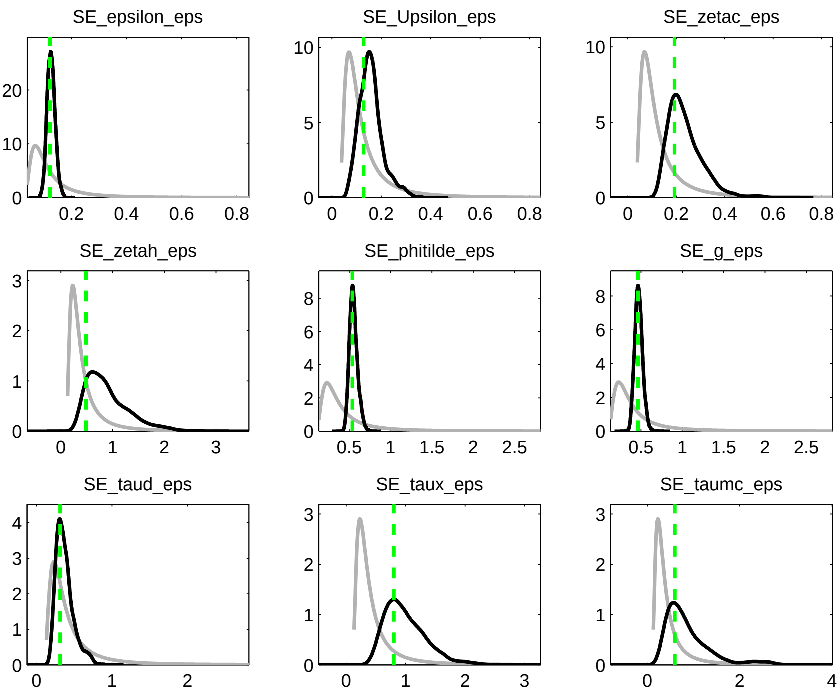

Fig. 19: Priors and posteriors.

Note: Financial frictions model. Prior distribution in gray, simulated distribution in black, and the computed posterior mode in dashed green.

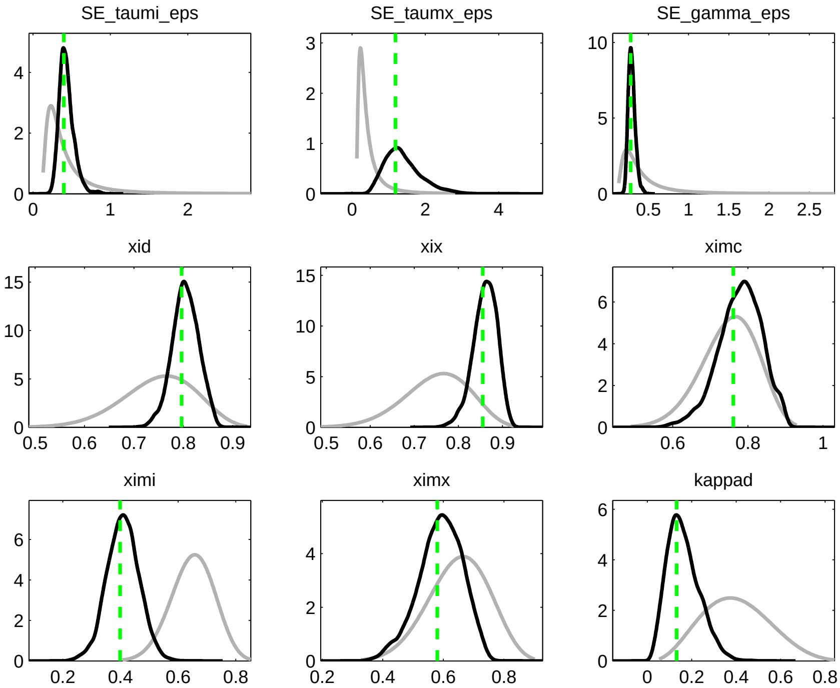

Fig. 20: Priors and posteriors (continued).

Note: Financial frictions model. Prior distribution in gray, simulated distribution in black, and the computed posterior mode in dashed green.

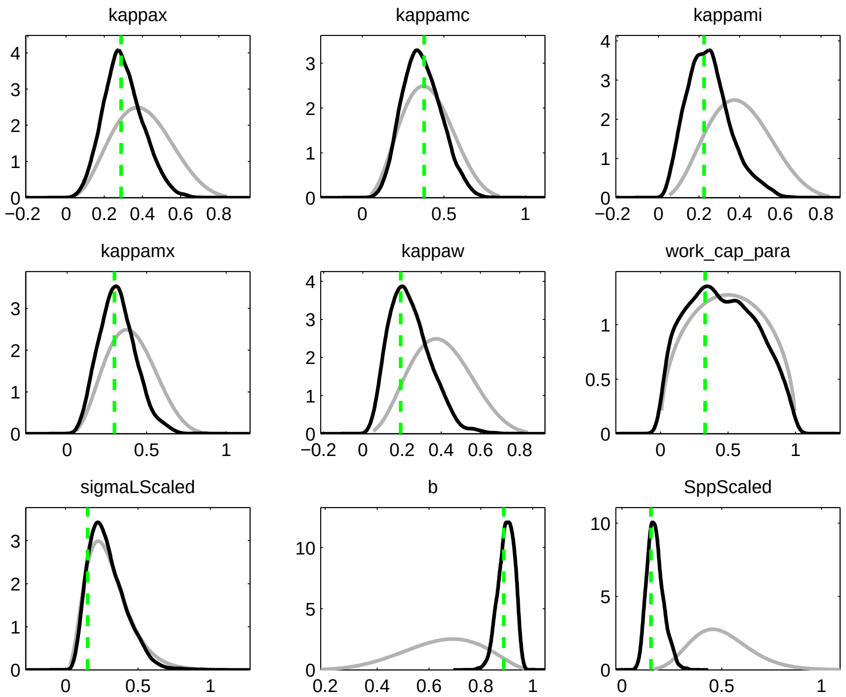

Fig. 21: Priors and posteriors (continued).

Note: Financial frictions model. Prior distribution in gray, simulated distribution in black, and the computed posterior mode in dashed green.

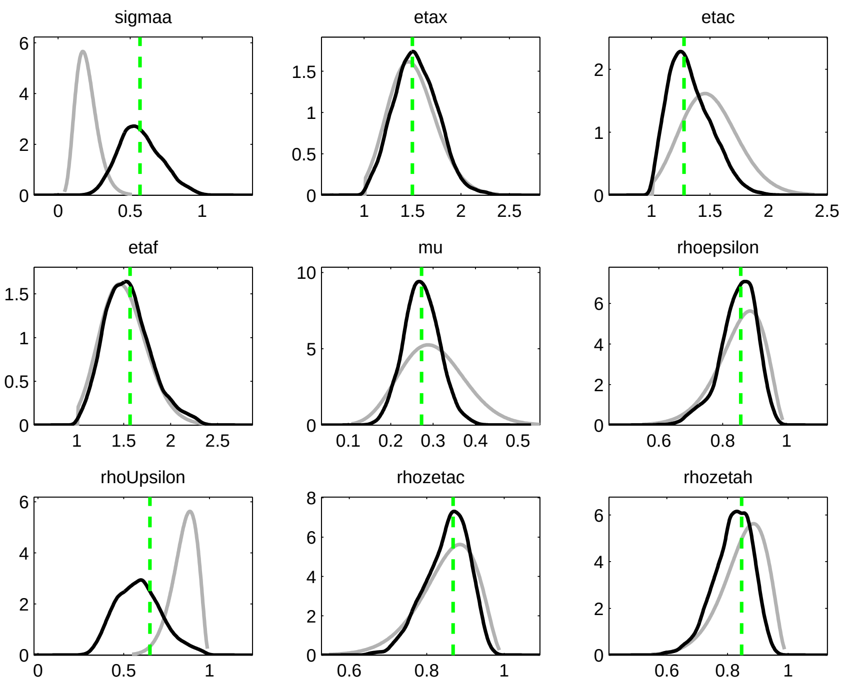

Fig. 22: Priors and posteriors (continued).

Note: Financial frictions model. Prior distribution in gray, simulated distribution in black, and the computed posterior mode in dashed green.

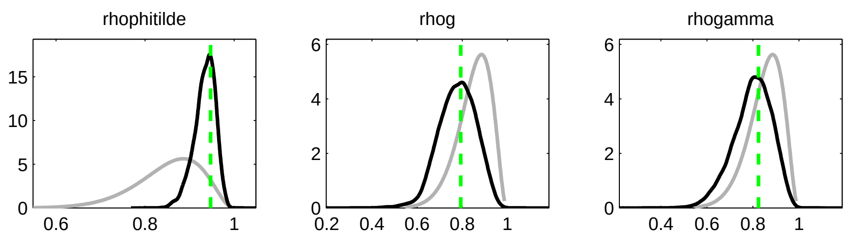

Fig. 23: Priors and posteriors (continued).

Note: Financial frictions model. Prior distribution in gray, simulated distribution in black, and the computed posterior mode in dashed green.

2 The model

2.1 The baseline model

As described in the main text, the three final goods - consumption, investment and exports - are produced by combining the domestic homogeneous good with specific imported inputs for each type of final good. Below we start the model description by going through the production of all these goods.

2.1.1 Production of the domestic homogeneous good

A homogeneous domestic good, Y_t, is produced using

$$Y_t = \left[\int_0^1 Y_{i,t}^{1/\lambda_d} di \right]^{\lambda_d}, \ 1 \leq \lambda_d < \infty, \tag{2.1}$$

where $Y_{i,t}$ denotes intermediate goods and $1/\lambda_d$ their degree of substitutability. The homogeneous domestic good is produced by a competitive, representative firm which takes the price of output, P_t, and the price of inputs, $P_{i,t}$, as given.

The i-th intermediate good producer has the following production function:

$$Y_{i,t} = (z_t H_{i,t})^{1-\alpha} \epsilon_t K_{i,t}^{\alpha} - z_t^{+} \phi, \tag{2.2}$$

where $K_{i,t}$ denotes the capital services rented by the i-th intermediate good producer. Also, $\log z_t$ is a technology shock whose first difference has a positive mean, $\log \epsilon_t$ is a stationary neutral technology shock and ϕ denotes a fixed production cost. The economy has two sources of growth: the positive drift in $\log z_t$ and a positive drift in $\log \Psi_t$, where Ψ_t is an investment-specific technology shock. The object z_t^{+} in (2.2) is defined as[1]

$$z_t^{+} = \Psi_t^{\frac{\alpha}{1-\alpha}} z_t.$$

In (2.2), $H_{i,t}$ denotes homogeneous labor services hired by the i-th intermediate good producer.

Firms must borrow a fraction ν^f of the wage bill, so that one unit of labor costs is denoted by

$$W_t R_t^f,$$

with

$$R_t^f = \nu^f R_t + 1 - \nu^f, \tag{2.3}$$

where W_t is the aggregate wage rate, and R_t is the risk-free interest rate that apply on working capital loans.

The firm's marginal cost, divided by the price of the homogeneous good is denoted by mc_t:

$$mc_t = \tau_t^d \left(\frac{1}{1-\alpha} \right)^{1-\alpha} \left(\frac{1}{\alpha} \right)^{\alpha} \left(r_t^k \right)^{\alpha} \left(\bar{w}_t R_t^f \right)^{1-\alpha} \frac{1}{\epsilon_t}, \tag{2.4}$$

where r_t^k is the nominal rental rate of capital scaled by P_t and $\bar{w}_t = W_t/(z_t^{+} P_t)$. Also, τ_t^d is a tax-like shock which affects marginal cost but does not appear in a production function.[2]

Productive efficiency dictates that marginal cost is equal to the cost of producing another unit using labor, implying:

$$mc_t = \tau_t^d \frac{\left(\mu_{\Psi,t} \right)^{\alpha} \bar{w}_t R_t^f}{\epsilon_t (1-\alpha) \left(\frac{k_{i,t}}{\mu_{z^+,t} H_{i,t}} \right)^{\alpha}} \tag{2.5}$$

The i-th firm is a monopolist in the production of the i-th good and so it sets its price. Price setting is subject to Calvo frictions. With probability ξ_d the intermediate good firm cannot reoptimize its price, in which case,

$$P_{i,t} = \tilde{\pi}_{d,t} P_{i,t-1},\ \tilde{\pi}_{d,t} := (\pi_{t-1})^{\kappa_d} (\bar{\pi}_t^c)^{1-\kappa_d-\varkappa_d} (\breve{\pi})^{\varkappa_d},$$

where κ_d, $\varkappa_d$, $\kappa_d + \varkappa_d \in (0,1)$ are parameters, π_{t-1} is the lagged inflation rate and $\bar{\pi}_t^c$ is the central bank's (implicit) target inflation rate. Also, $\breve{\pi}$ is a scalar which allowing to capture, among other things, the case in which non-optimizing firms either do not change price at all (i.e., $\breve{\pi} = \varkappa_d = 1$) or that they index only to the steady state inflation rate (i.e., $\breve{\pi} = \bar{\pi}$, $\varkappa_d = 1$). Note that there is a price dispersion in steady state if $\varkappa_d > 0$ and if $\breve{\pi}$ is different from the steady state value of π.

With probability $1 - \xi_d$ the firm can change the price. The problem of the i-th domestic intermediate good producer which has the opportunity to change price is to maximize discounted profits:

$$E_t \sum_{j=0}^{\infty} \beta^j \upsilon_{t+j} \left\{ P_{i,t+j} Y_{i,t+j} - mc_{t+j} P_{t+j} Y_{i,t+j} \right\}, \tag{2.6}$$

subject to the requirement that production equals demand. In the above expression, υ_t is the multiplier on the household's nominal budget constraint. It measures the marginal value to the household of one unit of profits in terms of currency. In states of nature when the firm can reoptimize price, it does so

[1] The details regarding the scaling of variables are collected in the online appendix section 3.

[2] In the linearized version of the model in which there are no price and wage distortions in the steady state, τ_t^d is isomorphic to a disturbance in λ_d, i.e., a markup shock.

to maximize its discounted profits subject to the price setting frictions and to the requirement that it satisfies demand given by

$$\left(\frac{P_t}{P_{i,t}}\right)^{\frac{\lambda_d}{\lambda_d-1}} Y_t = Y_{i,t}. \tag{2.7}$$

The equilibrium conditions associated with the price setting problem and their derivation are reported in the online appendix section 3.

The domestic intermediate output good is allocated among alternative uses as follows:

$$Y_t = G_t + C_t^d + I_t^d + \int_0^1 X_{i,t}^d di, \tag{2.8}$$

where G_t denotes government consumption (which consists entirely of the domestic good), C_t^d denotes intermediate goods used (together with foreign consumption goods) to produce final household consumption goods, I_t^d is the amount of intermediate domestic goods used in combination with imported foreign investment goods to produce a homogeneous investment good. Finally, the integral in (2.8) denotes domestic resources allocated to exports. The determination of consumption, investment and export demand is discussed below.

2.1.2 Production of final consumption and investment goods

Final consumption goods are purchased by households. These goods are produced by a representative competitive firm using the following linear homogeneous technology:

$$C_t = \left[(1-\omega_c)^{\frac{1}{\eta_c}} \left(C_t^d\right)^{\frac{\eta_c-1}{\eta_c}} + \omega_c^{\frac{1}{\eta_c}} \left(C_t^m\right)^{\frac{\eta_c-1}{\eta_c}}\right]^{\frac{\eta_c}{\eta_c-1}}. \tag{2.9}$$

The representative firm takes the price of final consumption goods output, P_t^c, as given. Final consumption goods output is produced using two inputs. The first, C_t^d, is a one-for-one transformation of the homogeneous domestic good and therefore has price, P_t. The second input, C_t^m, is the homogeneous composite of specialized consumption import goods discussed in the next subsection. The price of C_t^m is $P_t^{m,c}$. The representative firm takes the input prices, P_t and $P_t^{m,c}$ as given. Profit maximization leads to the following demand for the intermediate inputs in a scaled form:

$$\begin{aligned} c_t^d &= (1-\omega_c)(p_t^c)^{\eta_c} c_t \\ c_t^m &= \omega_c \left(\frac{p_t^c}{p_t^{m,c}}\right)^{\eta_c} c_t, \end{aligned} \tag{2.10}$$

where $p_t^c = P_t^c/P_t$ and $p_t^{m,c} = P_t^{m,c}/P_t$. The price of C_t is related to the price of the inputs by

$$p_t^c = \left[(1-\omega_c) + \omega_c (p_t^{m,c})^{1-\eta_c}\right]^{\frac{1}{1-\eta_c}}. \tag{2.11}$$

The rate of inflation of the consumption good is

$$\pi_t^c = \frac{P_t^c}{P_{t-1}^c} = \pi_t \left[\frac{(1-\omega_c) + \omega_c (p_t^{m,c})^{1-\eta_c}}{(1-\omega_c) + \omega_c (p_{t-1}^{m,c})^{1-\eta_c}}\right]^{\frac{1}{1-\eta_c}}. \tag{2.12}$$

Investment goods are produced by a representative competitive firm using the following technology:

$$I_t + a(u_t)\bar{K}_t = \Psi_t \left[(1-\omega_i)^{\frac{1}{\eta_i}} \left(I_t^d\right)^{\frac{\eta_i-1}{\eta_i}} + \omega_i^{\frac{1}{\eta_i}} \left(I_t^m\right)^{\frac{\eta_i-1}{\eta_i}}\right]^{\frac{\eta_i}{\eta_i-1}},$$

where investment is defined as the sum of investment goods, I_t, used in the accumulation of physical capital plus investment goods used in capital maintenance, $a(u_t)\bar{K}_t$. The maintenance is discussed below. See the online appendix section 3 for the functional form of $a(u_t)$. u_t denotes the utilization rate of capital, with capital services being defined by

$$K_t = u_t \bar{K}_t.$$

In order to accommodate the possibility that the price of investment goods relative to the price of consumption goods declines over time, it is assumed that the investment specific technology shock Ψ_t is a unit root process with a potentially positive drift. As in the consumption good sector, the representative investment goods producers take all relevant prices as given. Profit maximization leads to the following demand for the intermediate inputs in a scaled form:

$$i_t^d = \left(p_t^i\right)^{\eta_i} \left(i_t + a(u_t)\frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}}\right)(1-\omega_i) \tag{2.13}$$

$$i_t^m = \omega_i \left(\frac{p_t^i}{p_t^{m,i}}\right)^{\eta_i} \left(i_t + a(u_t)\frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}}\right) \tag{2.14}$$

where $p_t^i = \Psi_t P_t^i / P_t$ and $p_t^{m,i} = P_t^{m,i}/P_t$.

The price of I_t is related to the price of the inputs by

$$p_t^i = \left[(1-\omega_i) + \omega_i (p_t^{m,i})^{1-\eta_i}\right]^{\frac{1}{1-\eta_i}} . \tag{2.15}$$

The rate of inflation of the investment good is

$$\pi_t^i = \frac{\pi_t}{\mu_{\Psi,t}} \left[\frac{(1-\omega_i) + \omega_i \left(p_t^{m,i}\right)^{1-\eta_i}}{(1-\omega_i) + \omega_i \left(p_{t-1}^{m,i}\right)^{1-\eta_i}}\right]^{\frac{1}{1-\eta_i}} . \tag{2.16}$$

2.1.3 Exports and imports

Both exports and imports activities involve Calvo price setting frictions and therefore require the presence of market power. Dixit-Stiglitz strategy is used to introduce a range of specialized goods. This allows there to be market power without the counterfactual implication that there is a small number of firms in the export and import sector. Thus, exports involve a continuum of exporters, each of which is a monopolist which produces a specialized export good. Each monopolist produces the export good using a homogeneous domestically produced good and a homogeneous good derived from imports. The specialized export goods are sold to foreign competitive retailers which create a homogeneous good that is sold to foreign citizens.

In the case of imports, specialized domestic importers purchase a homogeneous foreign good which they turn into a specialized input and sell to domestic retailers. There are three types of domestic retailers. One uses the specialized import goods to create the homogeneous good used as an input into the production of specialized exports. Another uses the specialized import goods to create an input used in the production of investment goods. The third type uses specialized imports to produce a homogeneous input used in the production of consumption goods. Imported goods are combined with domestic inputs before being passed on to final domestic users. There are pricing frictions in both exports and imports. In all cases it is assumed that prices are set in the currency of the buyer ('pricing to market').[3]

Exports. There is a total demand by foreigners for domestic exports, which takes on the following form:

$$X_t = \left(\frac{P_t^x}{P_t^*}\right)^{-\eta_f} Y_t^*, \tag{2.17}$$

where Y_t^* is foreign GDP, P_t^* is the foreign currency price of foreign homogeneous goods and P_t^x is an index of export prices defined below. The goods X_t are produced by a representative competitive foreign retailer firm using specialized inputs as follows:

$$X_t = \left[\int_0^1 X_{i,t}^{\frac{1}{\lambda_x}} di\right]^{\lambda_x} , \tag{2.18}$$

[3] Pricing frictions in imports help the model account for the evidence that exchange rate shocks take time to pass into domestic prices. Pricing frictions in exports help the model to produce a hump-shape in the response of output to a domestic monetary shock, though, as seen in the main text, it is not the case for a currency area-wide monetary policy shock.

where $X_{i,t}$, $i \in (0,1)$ are specialized intermediate goods for export good production. The retailer that produces X_t takes its output price P_t^x and its input prices $P_{i,t}^x$ as given. Optimization leads to the following demand for specialized exports:

$$X_{i,t} = \left(\frac{P_{i,t}^x}{P_t^x}\right)^{\frac{-\lambda_x}{\lambda_x - 1}} X_t. \tag{2.19}$$

Combining (2.18) and (2.19),

$$P_t^x = \left[\int_0^1 \left(P_{i,t}^x\right)^{\frac{1}{1-\lambda_x}} di\right]^{1-\lambda_x}.$$

The i-th specialized export is produced by a monopolist using the following technology:

$$X_{i,t} = \left[\omega_x^{\frac{1}{\eta_x}} \left(X_{i,t}^m\right)^{\frac{\eta_x - 1}{\eta_x}} + (1-\omega_x)^{\frac{1}{\eta_x}} \left(X_{i,t}^d\right)^{\frac{\eta_x - 1}{\eta_x}}\right]^{\frac{\eta_x}{\eta_x - 1}},$$

where $X_{i,t}^m$ and $X_{i,t}^d$ are the i-th exporter's use of the imported and domestically produced goods, respectively. The marginal cost associated with the CES production function is derived from the multiplier associated with the Lagrangian representation of the cost minimization problem:

$$C = \min \tau_t^x \left[P_t^{m,x} R_t^x X_{i,t}^m + P_t R_t^x X_{i,t}^d\right] + \lambda \left\{X_{i,t} - \left[\omega_x^{\frac{1}{\eta_x}} \left(X_{i,t}^m\right)^{\frac{\eta_x - 1}{\eta_x}} + (1-\omega_x)^{\frac{1}{\eta_x}} \left(X_{i,t}^d\right)^{\frac{\eta_x - 1}{\eta_x}}\right]^{\frac{\eta_x}{\eta_x - 1}}\right\},$$

where $P_t^{m,x}$ is the price of the homogeneous import good and P_t is the price of the homogeneous domestic good. Using the first order conditions of this problem and the production function, the real marginal cost in terms of stationary variables, mc_t^x, is derived as

$$mc_t^x = \frac{\lambda}{S_t P_t^x} = \frac{\tau_t^x R_t^x}{q_t p_t^c p_t^x} \left[\omega_x \left(p_t^{m,x}\right)^{1-\eta_x} + (1-\omega_x)\right]^{\frac{1}{1-\eta_x}}, \tag{2.20}$$

where

$$R_t^x = \nu^x R_t + 1 - \nu^x, \tag{2.21}$$

$$\frac{S_t P_t^x}{P_t} = \frac{S_t P_t^*}{P_t^c} \frac{P_t^c}{P_t} \frac{P_t^x}{P_t^*} = q_t p_t^c p_t^x, \tag{2.22}$$

and q_t denotes the real exchange rate defined as

$$q_t = \frac{S_t P_t^*}{P_t^c}. \tag{2.23}$$

From the solution to the same problem, the demand for domestic inputs for export production is

$$X_{i,t}^d = \left(\frac{\lambda}{\tau_t^x R_t^x P_t}\right)^{\eta_x} X_{i,t}(1-\omega_x). \tag{2.24}$$

The quantity of the domestic homogeneous good used by specialized exporters is

$$\int_0^1 X_{i,t}^d di,$$

which, in terms of aggregates, is [by plugging (2.24) into this integral and derived in the online appendix section 3]

$$X_t^d = \int_0^1 X_{i,t}^d di = \left[\omega_x (p_t^{m,x})^{1-\eta_x} + (1-\omega_x)\right]^{\frac{\eta_x}{1-\eta_x}} (1-\omega_x)(\mathring{p}_t^x)^{\frac{-\lambda_x}{\lambda_x - 1}} (p_t^x)^{-\eta_f} Y_t^* \tag{2.25}$$

where $\mathring{p}_t^x$ is a measure of the price dispersion and is defined in the online appendix section 3.

Using a similar derivation as for X_t^d,

$$X_t^m = \omega_x \left(\frac{\left[\omega_x (p_t^{m,x})^{1-\eta_x} + (1-\omega_x)\right]^{\frac{1}{1-\eta_x}}}{p_t^{m,x}} \right)^{\eta_x} (\mathring{p}_t^x)^{\frac{-\lambda_x}{\lambda_x - 1}} (p_t^x)^{-\eta_f} Y_t^*. \tag{2.26}$$

The i-th, $i \in (0,1)$, export good firm takes (2.19) as its demand curve. This producer sets prices subject to a Calvo sticky-price mechanism. With probability ξ_x, the i-th export good firm cannot reoptimize its price, in which case it updates the price as follows:

$$P_{i,t}^x = \tilde{\pi}_t^x P_{i,t-1}^x, \ \tilde{\pi}_t^x = (\pi_{t-1}^x)^{\kappa_x} (\pi^x)^{1-\kappa_x - \varkappa_x} (\breve{\pi})^{\varkappa_x}, \tag{2.27}$$

where κ_x, $\varkappa_x$, $\kappa_x + \varkappa_x \in (0,1)$.

The equilibrium conditions associated with price setting by exporters that do get to reoptimize their prices are analogous to the ones derived for the domestic intermediate good producers and are reported in the online appendix section 3.

Imports. Foreign firms sell a homogeneous good to domestic importers. The importers convert the homogeneous good into a specialized input ('brand name it') and supply that input monopolistically to domestic retailers. Importers are subject to Calvo price setting frictions. There are three types of importing firms: (i) one produces goods used to produce an intermediate good for the production of consumption, (ii) one produces goods used to produce an intermediate good for the production of investment, and (iii) one produces goods used to produce an intermediate good for the production of exports.

Consider (i) first. The production function of the domestic retailer of imported consumption goods is

$$C_t^m = \left[\int_0^1 (C_{i,t}^m)^{\frac{1}{\lambda_{m,c}}} di \right]^{\lambda_{m,c}},$$

where $C_{i,t}^m$ is the output of the i-th specialized producer and C_t^m is an intermediate good used in the production of the consumption goods. Let $P_t^{m,c}$ denote the price index of C_t^m and let $P_{i,t}^{m,c}$ denote the price of the i-th intermediate input. The domestic retailer is competitive and takes $P_t^{m,c}$ and $P_{i,t}^{m,c}$ as given. The demand curve for specialized inputs is given by the domestic retailer's first order necessary condition for profit maximization:

$$C_{i,t}^m = C_t^m \left(\frac{P_t^{m,c}}{P_{i,t}^{m,c}} \right)^{\frac{\lambda_{m,c}}{\lambda_{m,c}-1}}.$$

We now turn to the producer of $C_{i,t}^m$ who takes the previous equation as a demand curve. This producer buys the homogeneous foreign good and converts it one-for-one into the domestic differentiated good, $C_{i,t}^m$. The intermediate good producer's marginal cost is

$$\tau_t^{m,c} S_t P_t^* R_t^{\nu,*}, \tag{2.28}$$

where

$$R_t^{\nu,*} = \nu^* R_t^* + 1 - \nu^*, \tag{2.29}$$

where R_t^* is the foreign nominal rate of interest.[4]

As in the homogeneous domestic good sector, $\tau_t^{m,c}$ is a tax-like shock which affects marginal costs but does not appear in a production function.[5]

The total value of imports accounted for by the consumption sector is

$$S_t P_t^* R_t^{\nu,*} C_t^m (\mathring{p}_t^{m,c})^{\frac{\lambda_{m,c}}{1-\lambda_{m,c}}},$$

[4] The notion here is that the intermediate good firm must pay the inputs with foreign currency and because they have no resources themselves at the beginning of the period, they must borrow those resources if they are to buy the foreign inputs needed to produce $C_{i,t}^m$. The financing need is in the foreign currency, so the loan is taken in that currency. There is no risk to this working capital loan because all shocks are realized at the beginning of the period and so there is no uncertainty within the duration of the loan about the realization of prices and exchange rates.

[5] In the linearization of a version of the model in which there are no price and wage distortions in the steady state, $\tau_t^{m,c}$ is isomorphic to a markup shock.

where

$$\mathring{p}_t^{m,c} = \frac{\mathring{P}_t^{m,c}}{P_t^{m,c}}$$

is a measure of the price dispersion in the differentiated good, $C_{i,t}^m$.

Now consider (ii). The production function for the domestic retailer of imported investment goods, I_t^m, is

$$I_t^m = \left[\int_0^1 (I_{i,t}^m)^{\frac{1}{\lambda_{m,i}}} di\right]^{\lambda_{m,i}}.$$

The retailer of imported investment goods is competitive and takes output prices, $P_t^{m,i}$, and input prices, $P_{i,t}^{m,i}$, as given.

The producer of the i-th intermediate input into the above production function buys the homogeneous foreign good and converts it one-for-one into the differentiated good, $I_{i,t}^m$. The marginal cost of $I_{i,t}^m$ is the analogue of (2.28):

$$\tau_t^{m,i} S_t P_t^* R_t^{\nu,*},$$

which implies the importing firm's cost is P_t^* (before borrowing costs, exchange rate conversion and markup shocks), which is the same cost for the specialized inputs used to produce C_t^m.

The total value of imports associated with the production of investment goods is analogous to what was obtained for the consumption good sector:

$$S_t P_t^* R_t^{\nu,*} I_t^m (\mathring{p}_t^{m,i})^{\frac{\lambda_{m,i}}{1-\lambda_{m,i}}},\ \mathring{p}_t^{m,i} = \frac{P_{i,t}^{m,i}}{P_t^{m,i}}. \tag{2.30}$$

Now consider (iii). The production function of the domestic retailer of imported goods used in the production of an input, X_t^m, for the production of export goods is

$$X_t^m = \left[\int_0^1 (X_{i,t}^m)^{\frac{1}{\lambda_{m,x}}} di\right]^{\lambda_{m,x}}.$$

The imported good retailer is competitive and takes output prices, $P_t^{m,x}$, and input prices, $P_{i,t}^{m,x}$, as given. The producer of the specialized input, $X_{i,t}^m$, has marginal cost

$$\tau_t^{m,x} S_t P_t^* R_t^{\nu,*}.$$

The total value of imports associated with the production of X_t^m is

$$S_t P_t^* R_t^{\nu,*} X_t^m (\mathring{p}_t^{m,x})^{\frac{\lambda_{m,x}}{1-\lambda_{m,x}}},\ \mathring{p}_t^{m,x} = \frac{P_{i,t}^{m,x}}{P_t^{m,x}}. \tag{2.31}$$

Each of the above three types of intermediate good firm is subject to Calvo price-setting frictions. With probability $1-\xi_{m,j}$, the j-th type of firm can reoptimize its price and with probability $\xi_{m,j}$ it updates its price according to

$$P_{i,t}^{m,j} = \tilde{\pi}_t^{m,j} P_{i,t-1}^{m,j},\ \tilde{\pi}_t^{m,j} := (\pi_{t-1}^{m,j})^{\kappa_{m,j}} (\bar{\pi}_t^c)^{1-\kappa_{m,j}-\varkappa_{m,j}} \breve{\pi}^{\varkappa_{m,j}},\ j = c, i, x. \tag{2.32}$$

The equilibrium conditions associated with price setting by importers are analogous to the ones derived for domestic intermediate good producers and are reported in the online appendix section 3.

2.1.4 Households

Household preferences are given by

$$E_0^j \sum_{t=0}^{\infty} \beta^t \left[\zeta_t^c \log(C_t - bC_{t-1}) - \zeta_t^h A_L \frac{(h_{j,t})^{1+\sigma_L}}{1+\sigma_L}\right], \tag{2.33}$$

where ζ_t^c denotes a consumption preference shock, ζ_t^h a disutility of labor shock, b is the consumption habit parameter, h_j denotes the j-th household's supply of labor services and σ_L denotes the inverse Frisch elasticity. The household owns the economy's stock of physical capital. It determines the rate at which the capital stock is accumulated and the rate at which it is utilized. The household also owns the stock of net foreign assets and determines its rate of accumulation.

Wage setting. The specialized labor supplied by households is combined by labor contractors into a homogeneous labor services:

$$H_t = \left[\int_0^1 (h_{j,t})^{\frac{1}{\lambda_w}} dj \right]^{\lambda_w}, \; 1 \leq \lambda_w < \infty.$$

Households are subject to Calvo wage setting frictions (as in Erceg, Henderson and Levin, 2000). With probability $1 - \xi_w$ the j-th household is able to reoptimize its wage and with probability ξ_w it updates its wage according to

$$W_{j,t+1} = \tilde{\pi}_{w,t+1} W_{j,t} \tag{2.34}$$

$$\tilde{\pi}_{w,t+1} = (\pi_t^c)^{\kappa_w} \left(\bar{\pi}_{t+1}^c\right)^{1-\kappa_w-\varkappa_w} (\breve{\pi})^{\varkappa_w} (\mu_{z^+})^{\vartheta_w}, \tag{2.35}$$

where κ_w, $\varkappa_w$, ϑ_w, $\kappa_w + \varkappa_w \in (0, 1)$.

Consider the j^{th} household that has an opportunity to reoptimize its wage at time t. Denote this wage rate by $\tilde{W}_t$. This is not indexed by j because the situation of each household that optimizes its wage is the same. In choosing $\tilde{W}_t$ the household considers the discounted utility (neglecting currently irrelevant terms in the household objective) of future histories when it cannot reoptimize (note the i vs j):

$$E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \left[-\zeta_{t+i}^h A_L \frac{(h_{j,t+i})^{1+\sigma_L}}{1+\sigma_L} + \upsilon_{t+i} W_{j,t+i} h_{j,t+i} \frac{1-\tau^y}{1+\tau^w} \right], \tag{2.36}$$

where τ_y is a tax on labor income, τ^w is a payroll tax, υ_t is the multiplier on the household's period t budget constraint. The demand for the j^{th} household's labor services, conditional on it having optimized in period t and not again since, is

$$h_{j,t+i} = \left(\frac{\tilde{W}_t \tilde{\pi}_{w,t+i}, \cdots, \tilde{\pi}_{w,t+1}}{W_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}, \tag{2.37}$$

where it is understood that $\tilde{\pi}_{w,t+i}, \cdots, \tilde{\pi}_{w,t+1} = 1$ when $i = 0$. The equilibrium conditions associated with this problem, i.e. wage setting of households that do get to reoptimize, are reported in the online appendix section 3.

Technology for capital accumulation. The law of motion of the stock of physical capital takes into account investment adjustment costs as introduced by Christiano, Eichenbaum and Evans (2005):[6]

$$\bar{K}_{t+1} = (1-\delta)\bar{K}_t + \Upsilon_t \left(1 - \tilde{S}\left(\frac{I_t}{I_{t-1}} \right) \right) I_t, \tag{2.38}$$

where Υ_t denotes the marginal efficiency of investment shock that affects how investment is transformed into capital.[7]

Household consumption and investment decisions. The first order condition for consumption is

$$\frac{\zeta_t^c}{c_t - bc_{t-1}\frac{1}{\mu_{z^+,t}}} - \beta b E_t \frac{\zeta_{t+1}^c}{c_{t+1}\mu_{z^+,t+1} - bc_t} - \psi_{z^+,t} p_t^c (1+\tau^c) = 0, \tag{2.39}$$

where

$$\psi_{z^+,t} = \upsilon_t P_t z_t^+$$

is the marginal value of wealth in real terms, in particular in terms of one unit of the homogeneous domestic good at time t.

[6] See the online appendix section 3 for the functional form of the investment adjustment costs, $\tilde{S}$.
[7] This is the shock whose importance is emphasized by Justiniano, Primiceri and Tambalotti (2011).

To define the intertemporal Euler equation associated with the household's capital accumulation decision, define the rate of return on a period t investment in a unit of physical capital, R^k_{t+1}:

$$R^k_{t+1} = \frac{(1-\tau^k)\left[u_{t+1}r^k_{t+1} - \frac{p^i_{t+1}}{\Psi_{t+1}}a(u_{t+1})\right]P_{t+1} + (1-\delta)P_{t+1}P_{k',t+1} + \tau^k\delta P_t P_{k',t}}{P_t P_{k',t}}, \tag{2.40}$$

where

$$\frac{p^i_t}{\Psi_t}P_t = P^i_t$$

is the date t price of the homogeneous investment good, $\bar{r}^k_t = \Psi_t r^k_t$ is the scaled real rental rate of capital, τ^k is the capital tax rate, $P_{k',t}$ denotes the price of a unit of newly installed physical capital which operates in period $t+1$. This price is expressed in units of the homogeneous good, so that $P_t P_{k',t}$ is the domestic currency price of physical capital. The numerator in the expression for R^k_{t+1} represents the period $t+1$ payoff from a unit additional physical capital. The expression in square brackets captures the idea that maintenance expenses associated with the operation of capital are deductible from taxes. The last expression in the numerator expresses the idea that physical depreciation is deductible at historical cost. It is convenient to express R^k_t in scaled terms:

$$R^k_{t+1} = \frac{\pi_{t+1}}{\mu_{\Psi,t+1}} \frac{(1-\tau^k)\left[u_{t+1}\bar{r}^k_{t+1} - p^i_{t+1}a(u_{t+1})\right] + (1-\delta)p_{k',t+1} + \tau^k\delta\frac{\mu_{\Psi,t+1}}{\pi_{t+1}}p_{k',t}}{p_{k',t}}, \tag{2.41}$$

where $p_{k',t} = \Psi_t P_{k',t}$.[8] The first order condition for capital implies

$$\psi_{z^+,t} = \beta E_t \psi_{z^+,t+1}\frac{R^k_{t+1}}{\pi_{t+1}\mu_{z^+,t+1}}. \tag{2.42}$$

By differentiating the Lagrangian representation of the household's problem with respect to I_t, the investment first order condition in scaled terms is

$$-\psi_{z^+,t}p^i_t + \psi_{z^+,t}p_{k',t}\Upsilon_t\left[1 - \tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right) - \tilde{S}'\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right]$$
$$+ \beta\psi_{z^+,t+1}p_{k',t+1}\Upsilon_{t+1}\tilde{S}'\left(\frac{\mu_{z^+,t+1}\mu_{\Psi,t+1}i_{t+1}}{i_t}\right)\left(\frac{i_{t+1}}{i_t}\right)^2\mu_{\Psi,t+1}\mu_{z^+,t+1} = 0. \tag{2.43}$$

The first order condition associated with capital utilization is, in scaled terms[9]

$$\bar{r}^k_t = p^i_t a'(u_t). \tag{2.44}$$

Financial assets. The household does the domestic economy's saving. Period t saving occurs by the acquisition of net foreign assets, A^*_{t+1}, and a domestic asset. The domestic asset is used to finance the working capital requirements of firms. This asset pays a nominally non-state contingent return from t to $t+1$, R_t. The first order condition associated with this domestic asset is

$$\psi_{z^+,t} = \beta E_t \frac{\psi_{z^+,t+1}}{\mu_{z^+,t+1}}\left[\frac{R_t - \tau^b(R_t - \pi_{t+1})}{\pi_{t+1}}\right], \tag{2.45}$$

where τ^b is the tax rate on the real interest rate on bond income.[10]

The tax treatment of domestic agent's earnings on foreign bonds is the same as the tax treatment of agent's earnings on domestic bonds. The date t first order condition associated with the asset A^*_{t+1} that pays R^*_t in terms of foreign currency is

$$\upsilon_t S_t = \beta E_t \upsilon_{t+1}\left[S_{t+1}R^*_t\Phi_t - \tau^b\left(S_{t+1}R^*_t\Phi_t - \frac{S_t}{P_t}P_{t+1}\right)\right]. \tag{2.46}$$

[8] A rise in inflation raises the tax rate on capital because of the practice of valuing depreciation at historical cost.

[9] The tax rate on capital income does not enter here because of the deductibility of maintenance costs.

[10] A consequence of this treatment of the taxation on domestic bonds is that the steady state real after-tax return on bonds is invariant to π.

Recall that S_t is the domestic currency price of a unit foreign currency. The left side of this expression is the cost of acquiring a unit of foreign assets. The currency cost is S_t and this is converted into utility terms by multiplying by the multiplier on the household's budget constraint, υ_t. The term in square brackets is the after-tax payoff of the foreign asset in domestic currency units. The period $t+1$ pre-tax interest payoff on A^*_{t+1} is $S_{t+1}R^*_t\Phi_t$. Here, R^*_t is the foreign nominal rate of interest, which is risk free in foreign currency units. The term Φ_t represents a relative risk adjustment of the foreign asset return, so that a unit of the foreign asset acquired in t pays off $R^*_t\Phi_t$ units of foreign currency in $t+1$. The determination of Φ_t is discussed below. The remaining term in brackets pertains to the impact of taxation on returns on foreign assets.[11]

Scaling the first order condition, (2.46), by multiplying both sides by $P_t z^+_t/S_t$ yields

$$\psi_{z^+,t} = \beta E_t \frac{\psi_{z^+,t+1}}{\pi_{t+1}\mu_{z^+,t+1}} \left[s_{t+1}R^*_t\Phi_t - \tau^b(s_{t+1}R^*_t\Phi_t - \pi_{t+1})\right], \tag{2.47}$$

where

$$s_t = \frac{S_t}{S_{t-1}}.$$

The risk adjustment term has the following form:

$$\Phi_t = \Phi\left(a_t, R^*_t - R_t, \tilde{\phi}_t\right) = \exp\left(-\tilde{\phi}_a(a_t - \bar{a}) - \tilde{\phi}_s(R^*_t - R_t - (R^* - R)) + \tilde{\phi}_t\right), \tag{2.48}$$

where

$$a_t = \frac{S_t A^*_{t+1}}{P_t z^+_t},$$

$\tilde{\phi}_t$ is a mean zero country risk premium shock, and $\tilde{\phi}_a$ and $\tilde{\phi}_s$ are positive parameters.[12]

2.1.5 Fiscal and monetary authorities

The monetary policy is conducted according to a hard peg of the domestic nominal interest rate to the foreign nominal interest rate.

Government expenditures are modeled as

$$G_t = g_t z^+_t,$$

where g_t is an exogenous stochastic process, and z^+_t ensures a constant government expenditures to GDP ratio. The tax rates in the model are: capital tax rate, τ^k, bond tax rate, τ^b, labor income tax rate, τ^y, consumption tax rate, τ^c, and payroll tax rate, τ^w. Any difference between government expenditures and tax revenues is offset by lump-sum transfers.

2.1.6 Foreign variables

The representation of foreign variables takes into account the assumption that foreign output, Y^*_t, is affected by disturbances to z^+_t, just as domestic variables are. In particular,

$$\begin{aligned}\log Y^*_t &= \log y^*_t + \log z^+_t \\ &= \log y^*_t + \log z_t + \frac{\alpha}{1-\alpha}\log\psi_t,\end{aligned}$$

[11] If we ignore the term after the minus sign in parentheses, then the taxation is applied to the whole nominal payoff on the bond, including principal. The term after the minus sign is designed to ensure that the principal is deducted from taxes. The principal is expressed in nominal terms and is set so that the real value at $t+1$ coincides with the real value of the currency used to purchase the asset in period t. Recall that S_t is the period t domestic currency cost of a unit (in terms of foreign currency) of foreign assets. So the period t real cost of the asset is S_t/P_t. The domestic currency value in period $t+1$ of this real quantity is $P_{t+1}S_t/P_t$.

[12] The dependence of Φ_t on a_t ensures that there is a unique steady state value of a_t that is independent of the initial net foreign assets and the capital stock of the economy. The dependence of Φ_t on the relative level of the interest rate, $R^*_t - R_t$, is designed to allow the model to reproduce two types of observations. The first concerns observations related to uncovered interest parity. The second concerns the hump-shaped response of output to a domestic monetary policy shock. The particular calibration sets $\tilde{\phi}_s = 0$ to ensure the nominal interest rate peg regime.

where $\log(y_t^*)$ is assumed to be a stationary process. It is assumed that

$$\begin{pmatrix} \log\left(\frac{y_t^*}{y^*}\right) \\ \pi_t^* - \pi^* \\ R_t^* - R^* \\ \log\left(\frac{\mu_{z,t}}{\mu_z}\right) \\ \log\left(\frac{\mu_{\psi,t}}{\mu_\psi}\right) \end{pmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & \frac{a_{24}\alpha}{1-\alpha} \\ a_{31} & a_{32} & a_{33} & a_{34} & \frac{a_{34}\alpha}{1-\alpha} \\ 0 & 0 & 0 & \rho_{\mu_z} & 0 \\ 0 & 0 & 0 & 0 & \rho_{\mu_\psi} \end{bmatrix} \begin{pmatrix} \log\left(\frac{y_{t-1}^*}{y^*}\right) \\ \pi_{t-1}^* - \pi^* \\ R_{t-1}^* - R^* \\ \log\left(\frac{\mu_{z,t-1}}{\mu_z}\right) \\ \log\left(\frac{\mu_{\psi,t-1}}{\mu_\psi}\right) \end{pmatrix}$$

$$+ \begin{bmatrix} \sigma_{y^*} & 0 & 0 & 0 & 0 \\ c_{21} & \sigma_{\pi^*} & 0 & c_{24} & \frac{c_{24}\alpha}{1-\alpha} \\ c_{31} & c_{32} & \sigma_{R^*} & c_{34} & \frac{c_{34}\alpha}{1-\alpha} \\ 0 & 0 & 0 & \sigma_{\mu_z} & 0 \\ 0 & 0 & 0 & 0 & \sigma_{\mu_\psi} \end{bmatrix} \begin{pmatrix} \varepsilon_{y^*,t} \\ \varepsilon_{\pi^*,t} \\ \varepsilon_{R^*,t} \\ \varepsilon_{\mu_z,t} \\ \varepsilon_{\mu_\psi,t} \end{pmatrix},$$

where ε_t's are mean zero, unit variance, Gaussian i.i.d. processes uncorrelated with each other.

In matrix form,

$$X_t^* = AX_{t-1}^* + C\varepsilon_t$$

in obvious notation. Note that the matrix C has 10 elements, so that the order condition for identification is satisfied, since $C'C$ represents 15 independent equations. The above restrictions assume that the shock $\varepsilon_{y^*,t}$ affects the first three variables in X_t^*, while $\varepsilon_{\pi^*,t}$ only affects the second two, and $\varepsilon_{R^*,t}$ only affects the third.[13] Also, the zeros in the last two columns of the first row in A and C imply that the technology shocks do not affect y_t^*.[14] Third, the A and C matrices capture the notion that innovations to technology affect foreign inflation and the interest rate via their impact on z_t^+. Fourth, the assumptions on A and C imply that $\log\left(\frac{\mu_{\psi,t}}{\mu_\psi}\right)$ and $\log\left(\frac{\mu_{z,t}}{\mu_z}\right)$ are univariate first order autoregressive processes driven by $\epsilon_{\mu_\psi,t}$ and $\varepsilon_{\mu_z,t}$, respectively.

2.1.7 Resource constraints

The fact that there is potentially steady state price dispersion both in prices and wages complicates the expression for the domestic homogeneous good, Y_t, in terms of aggregate factors of production. The relationship derived in the online appendix section 3 can be expressed as

$$y_t = (\mathring{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} \left[\epsilon_t \left(\frac{1}{\mu_{\Psi,t}} \frac{1}{\mu_{z^*,t}} k_t \right)^\alpha \left(\mathring{w}_t^{-\frac{\lambda_w}{1-\lambda_w}} h_t \right)^{1-\alpha} - \phi \right], \tag{2.49}$$

where $\mathring{p}_t$ denotes the degree of price dispersion in the intermediate domestic good.

Resource constraint for domestic homogeneous output. Above we defined real, scaled output in terms of aggregate factors of production. It is convenient to also have an expression that exhibits the uses of domestic homogeneous output. Using (2.25),

$$z_t^+ y_t = G_t + C_t^d + I_t^d + \left[\omega_x \left(p_t^{m,x}\right)^{1-\eta_x} + (1-\omega_x) \right]^{\frac{\eta_x}{1-\eta_x}} (1-\omega_x) (\mathring{p}_t^x)^{\frac{-\lambda_x}{\lambda_x - 1}} (p_t^x)^{-\eta_f} Y_t^*,$$

or, after scaling by z_t^+ and using (2.10)

$$\begin{aligned} y_t =& g_t + (1-\omega_c)(p_t^c)^{\eta_c} c_t + \left(p_t^i\right)^{\eta_i} \left(i_t + a(u_t) \frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}} \right) (1-\omega_i) \\ &+ \left[\omega_x (p_t^{m,x})^{1-\eta_x} + (1-\omega_x) \right]^{\frac{\eta_x}{1-\eta_x}} (1-\omega_x)(\mathring{p}_t^x)^{\frac{-\lambda_x}{\lambda_x - 1}} (p_t^x)^{-\eta_x} y_t^*. \end{aligned} \tag{2.50}$$

When GDP is matched to the data, capital utilization costs are subtracted from y_t (see the online appendix section 3):

$$gdp_t = y_t - (p_t^i)^{\eta_i} \left(a(u_t) \frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}} \right) (1-\omega_i).$$

[13] The assumption about $\varepsilon_{R^*,t}$ corresponds to one strategy for identifying a monetary policy shock, in which it is assumed that inflation and output are predetermined relative to the monetary policy shock.

[14] This reflects the assumption that the impact of technology shocks on Y_t^* is completely taken into account by z_t^+, while other shocks to Y_t^* are orthogonal to z_t^+ and they affect Y_t^* via y_t^*.

Trade balance. Expenses on imports and new purchases of net foreign assets, A^*_{t+1}, must equal income from exports and from previously purchased net foreign assets:

$$S_t A^*_{t+1} + expenses\, on\, imports_t = receipts\, from\, exports_t + R^*_{t-1}\Phi_{t-1}S_t A^*_t.$$

Expenses on imports correspond to the purchases of specialized importers for the consumption, investment and export sectors:[15]

$$expenses\, on\, imports_t = S_t P^*_t R^{\nu,*}_t \left(C^m_t (\mathring{p}^{m,c}_t)^{\frac{\lambda_{m,c}}{1-\lambda_{m,c}}} + I^m_t (\mathring{p}^{m,i}_t)^{\frac{\lambda_{m,i}}{1-\lambda_{m,i}}} + X^m_t (\mathring{p}^{m,x}_t)^{\frac{\lambda_{m,x}}{1-\lambda_{m,x}}} \right).$$

The current account can be written as follows in scaled form, using (2.22):

$$a_t + q_t p^c_t R^{\nu,*}_t \left(c^m_t (\mathring{p}^{m,c}_t)^{\frac{\lambda_{m,c}}{1-\lambda_{m,c}}} + i^m_t (\mathring{p}^{m,i}_t)^{\frac{\lambda_{m,i}}{1-\lambda_{m,i}}} + x^m_t (\mathring{p}^{m,x}_t)^{\frac{\lambda_{m,x}}{1-\lambda_{m,x}}} \right)$$
$$= q_t p^c_t p^x_t x_t + R^*_{t-1}\Phi_{t-1} s_t \frac{a_{t-1}}{\pi_t \mu_{z^+,t}}, \qquad (2.51)$$

where $a_t = S_t A^*_{t+1}/(P_t z^+_t)$.

This completes the description of the baseline model. Additional equilibrium conditions and the complete list of endogenous variables are in the the online appendix section 3.

2.2 Financial frictions in the model

2.2.1 Overview of the financial frictions model

A number of the activities in the baseline model require financing. Producers of specialized inputs must borrow working capital within the period. The management of capital involves financing because the construction of capital requires a substantial initial outlay of resources, while the return from capital comes in over time as a flow. In the baseline model financing requirements affect the allocations, but not very much. This is because none of the messy realities of actual financial markets are present. There is no asymmetric information between borrower and lender, there is no risk to lenders. In the case of capital accumulation, the borrower and lender are actually the same household who puts up finances and later reaps the rewards. When real-world financial frictions are introduced into the model, then intermediation becomes distorted by the presence of balance sheet constraints and other factors.

This subsection assumes that the accumulation and management of capital involves frictions following Bernanke, Gertler and Gilchrist (1999) (henceforth, BGG). It is assumed that working capital loans are frictionless.

Recall that households deposit money with banks, and that the interest rate that households receive is nominally non state-contingent. This gives rise to potentially interesting wealth effects of the sort emphasized by Fisher (1933). The banks then lend funds to entrepreneurs using a standard nominal debt contract which is optimal given the asymmetric information. The amount that banks are willing to lend to an entrepreneur under the standard debt contract is a function of the entrepreneur's net worth. This is how balance sheet constraints enter the model. When a shock occurs that reduces the value of the entrepreneur's assets, this cuts into their ability to borrow. As a result, they acquire less capital and this translates into a reduction in investment and ultimately into a slowdown in the economy.

Although individual entrepreneurs are risky, banks themselves are not. It is supposed that banks lend to a sufficiently diverse group of entrepreneurs that the uncertainty that exists in individual entrepreneurial loans washes out across all loans. The net worth of entrepreneurs is empirically measured by using a stock market index.

Entrepreneurs all have different histories, as they experience different idiosyncratic shocks. Thus, in general, solving for the aggregate variables would require also solving for the distribution of entrepreneurs

[15] Note the presence of the price distortion terms here. To understand these terms, recall that, e.g., C^m_t is produced as a linear homogeneous function of specialized imported goods. Because the specialized importers only buy foreign goods, it is their total expenditures that interests us here. When the imports are distributed evenly across differentiated goods, then the total quantity of those imports is C^m_t, and the value of imports associated with domestic production of consumption goods is $S_t P^*_t R^{\nu,*}_t C^m_t$. When there are price distortion among imported intermediate goods then the sum of the homogeneous import goods is higher for any given value of C^m_t.

according to their characteristics and for the law of motion for that distribution. However, as emphasized in BGG, the right functional form assumption have been made in the model to guarantee the result that the aggregate variables associated with entrepreneurs are not a function of distributions. The loan contract specifies that all entrepreneurs, regardless of their net worth, receive the same interest rate. Also, the loan amount received by an entrepreneur is proportional to his level of net worth. These characteristics are enough to guarantee the aggregation result. The financial frictions bring a net increase of two equations over the equations in the baseline model. In addition, they introduce two new endogenous variables, one related to the interest rate paid by entrepreneurs and the other to their net worth. The financial frictions also allow to introduce two new shocks. A formal discussion of the model follows.

2.2.2 The individual entrepreneur

At the end of period t each entrepreneur has a level of net worth, N_{t+1}. The entrepreneur's net worth, N_{t+1}, constitutes his state at this time, and nothing else about his history is relevant. There are many entrepreneurs for each level of net worth and for each level of net worth there is a competitive bank with free entry that offers a loan contract. The contract is defined by a loan amount and by an interest rate, both of which are derived as the solution to a particular optimization problem.

Consider a type of entrepreneur with particular level of net worth, N_{t+1}. The entrepreneur combines this net worth with a bank loan, B_{t+1}, to purchase new installed physical capital, $\bar{K}_{t+1}$, from capital producers. The loan the entrepreneur requires for this is

$$B_{t+1} = P_t P_{k',t} \bar{K}_{t+1} - N_{t+1}. \tag{2.52}$$

The entrepreneur is required to pay a gross interest rate, Z_{t+1}, on the bank loan at the end of period $t+1$, if it is feasible to do so. After purchasing capital, the entrepreneur experiences an idiosyncratic productivity shock which converts the purchased capital, $\bar{K}_{t+1}$, into $\bar{K}_{t+1}\omega$, where ω is a unit mean, log-normally and independently distributed random variable across entrepreneurs with $V(\log \omega) = \sigma_t^2$. The t subscript indicates that σ_t is itself the realization of a random variable. This allows to consider the effects of an increase in the riskiness of individual entrepreneurs and we call σ_t the shock to idiosyncratic uncertainty. Denote the cumulative distribution function of ω by $F(\omega;\sigma)$ and its partial derivatives by $F_\omega(\omega;\sigma)$ and $F_\sigma(\omega;\sigma)$.

After observing the period $t+1$ shocks, the entrepreneur sets the utilization rate, u_{t+1}, of capital and rents out capital in competitive markets at the nominal rental rate, $P_{t+1}r_{t+1}^k$. In choosing the capital utilization rate, the entrepreneur takes into account that operating one unit of physical capital at rate u_{t+1} requires $a(u_{t+1})$ of domestically produced investment goods for maintenance expenditures, where a is defined in the online appendix section 3. The entrepreneur then sells the undepreciated part of physical capital to capital producers. Per unit of physical capital purchased, the entrepreneur who draws idiosyncratic productivity ω earns a return (after taxes) of $R_{t+1}^k\omega$, where R_{t+1}^k is defined in (2.40). Because the mean of ω across entrepreneurs is unity, the average return across all entrepreneurs is R_{t+1}^k.

After entrepreneurs sell their capital, they settle their bank loans. At this point the resources available to an entrepreneur who has purchased $\bar{K}_{t+1}$ units of physical capital in period t and who experiences an idiosyncratic productivity shock ω are $P_t P_{k',t} R_{t+1}^k \omega \bar{K}_{t+1}$. There is a cutoff value of ω, $\bar{\omega}_{t+1}$, such that the entrepreneur has just enough resources to pay interest:

$$\bar{\omega}_{t+1} R_{t+1}^k P_t P_{k',t} \bar{K}_{t+1} = Z_{t+1} B_{t+1}. \tag{2.53}$$

Entrepreneurs with $\omega < \bar{\omega}_{t+1}$ are bankrupt and turn over all their resources,

$$R_{t+1}^k \omega P_t P_{k',t} \bar{K}_{t+1},$$

which is less than $Z_{t+1}B_{t+1}$, to the bank. In this case, the bank monitors the entrepreneur at the cost

$$\mu R_{t+1}^k \omega P_t P_{k',t} \bar{K}_{t+1},$$

where $\mu \geq 0$ is a parameter.

Banks obtain the funds loaned in the period t to entrepreneurs by issuing deposits to households at gross nominal rate of interest, R_t. The subscript on R_t indicates that the payoff to households in $t+1$ is not contingent on the period $t+1$ uncertainty. There is no risk in household bank deposits, and the household Euler equation associated with deposits is exactly the same as in (2.45).

There is competition and free entry among banks and banks participate in no financial arrangements other than the liabilities issued to households and the loans issued to entrepreneurs. It follows that the bank's cash flow in each state of period $t+1$ is zero for each loan amount.[16] For loans in the amount B_{t+1}, the bank receives gross interest $Z_{t+1}B_{t+1}$ from the fraction $1-F(\bar{\omega}_{t+1};\sigma_t)$ of entrepreneurs who are not bankrupt. The bank takes all the resources possessed by bankrupt entrepreneurs, net of monitoring costs. Thus, the state-by-state zero profit condition is

$$[1-F(\bar{\omega}_{t+1};\sigma_t)]Z_{t+1}B_{t+1}+(1-\mu)\int_0^{\bar{\omega}_{t+1}}\omega dF(\omega;\sigma_t)R^k_{t+1}P_tP_{k',t}\bar{K}_{t+1}=R_tB_{t+1},$$

or, after making use of (2.53) and rearranging,

$$[\Gamma(\bar{\omega}_{t+1};\sigma_t)-\mu G(\bar{\omega}_{t+1};\sigma_t)]\frac{R^k_{t+1}}{R_t}\varrho_t=\varrho_t-1, \tag{2.54}$$

where

$$\begin{aligned} G(\bar{\omega}_{t+1};\sigma_t) &= \int_0^{\bar{\omega}_{t+1}}\omega dF(\omega;\sigma_t) \\ \Gamma(\bar{\omega}_{t+1};\sigma_t) &= \bar{\omega}_{t+1}[1-F(\bar{\omega}_{t+1};\sigma_t)]+G(\bar{\omega}_{t+1};\sigma_t) \\ \varrho_t &= \frac{P_tP_{k',t}\bar{K}_{t+1}}{N_{t+1}}. \end{aligned}$$

The expression $\Gamma(\bar{\omega}_{t+1};\sigma_t)-\mu G(\bar{\omega}_{t+1};\sigma_t)$ is the share of revenues earned by entrepreneurs that borrow B_{t+1} which goes to banks. Note that $\Gamma_{\bar{\omega}}(\bar{\omega}_{t+1};\sigma_t)=1-F(\bar{\omega}_{t+1};\sigma_t)>0$ and $G_{\bar{\omega}}(\bar{\omega}_{t+1};\sigma_t)=\bar{\omega}_{t+1}F_{\bar{\omega}}(\bar{\omega}_{t+1};\sigma_t)>0$. Therefore, the share of entrepreneurial revenues accruing to banks is non-monotone with respect to $\bar{\omega}_{t+1}$.[17]

The optimal contract is derived in the online appendix section 3. ϱ_t and $\bar{\omega}_{t+1}$ are the same for all entrepreneurs regardless of their net worth. This result of the leverage ratio, ϱ_t, implies that

$$\frac{B_{t+1}}{N_{t+1}}=\varrho_t-1,$$

i.e., that an entrepreneur's loan amount is proportional to his net worth. Rewriting (2.52) and (2.53), the rate of interest paid by the entrepreneur is

$$Z_{t+1}=\frac{\bar{\omega}_{t+1}R^k_{t+1}}{1-\frac{N_{t+1}}{P_tP_{k',t}\bar{K}_{t+1}}}=\frac{\bar{\omega}_{t+1}R^k_{t+1}}{1-\frac{1}{\varrho_t}}, \tag{2.55}$$

which also is the same for all entrepreneurs regardless of their net worth.

---

[16] Absence of state contingent securities markets guarantee that cash flow is non-negative. Free entry guarantees that ex ante profits are zero. Given that each state of nature receives positive probability, the two assumptions imply the state-by-state zero profit condition.

[17] BGG argue that the expression on the left of (2.54) has an inverted 'U' shape, achieving a maximum value at $\bar{\omega}_{t+1}=\omega^*$. The expression is increasing for $\bar{\omega}_{t+1}<\omega^*$ and decreasing for $\bar{\omega}_{t+1}>\omega^*$. Thus, for any given value of the leverage ratio, ϱ_t, and R^k_{t+1}/R_t, there are either no values of $\bar{\omega}_{t+1}$ or two that satisfy (2.54). The value of $\bar{\omega}_{t+1}$ realized in equilibrium must be the one on the left side of inverted 'U' shape. This is because, according to (2.53), the lower value of $\bar{\omega}_{t+1}$ corresponds to a lower interest rate for entrepreneurs which yields them higher welfare. The equilibrium contract is the one that maximizes entrepreneurial welfare subject to the zero profit condition on banks. This reasoning leads to the conclusion that $\bar{\omega}_{t+1}$ falls with a period $t+1$ shock that drives R^k_{t+1} up. The fraction of entrepreneurs that experience bankruptcy is $F(\bar{\omega}_{t+1};\sigma_t)$, so it follows that a shock which drives up R^k_{t+1} has a negative contemporaneous impact on the bankruptcy rate. According to (2.40), shocks that drive R^k_{t+1} up include anything which raises the value of physical capital and/or the rental rate of capital.

2.2.3 Aggregation across entrepreneurs and the external financing premium

The law of motion for the net worth on an individual entrepreneur is

$$V_t = R_t^k P_{t-1} P_{k',t-1} K_t - \Gamma(\bar{\omega}_t; \sigma_{t-1}) R_t^k P_{t-1} P_{k',t-1} K_t.$$

Each entrepreneur faces an identical and independent probability $1 - \gamma_t$ of being selected to exit the economy. With the probability γ_t each entrepreneur remains. Because the selection is random, the net worth of the entrepreneurs who survive is $\gamma_t \bar{V}_t$. A fraction $1-\gamma_t$ of new entrepreneurs arrive. Entrepreneurs who survive or who are new arrivals receive a transfer W_t^e. This ensures that all entrepreneurs, whether new arrivals or survivors that experienced bankruptcy, have sufficient funds to obtain at least some amount of loans. The average net worth across all entrepreneurs after the W_t^e transfers have been made and exits and entry have occurred, is $\bar{N}_{t+1} = \gamma_t \bar{V}_t + W_t^e$, or

$$\begin{aligned}\bar{N}_{t+1} =& \gamma_t \Bigg\{ R_t^k P_{t-1} P_{k',t-1} \bar{K}_t - \left[R_{t-1} + \frac{\mu \int_0^{\bar{\omega}_t} \omega dF(\omega; \sigma_{t-1}) R_t^k P_{t-1} P_{k',t-1} \bar{K}_t}{P_{t-1} P_{k',t-1} \bar{K}_t - \bar{N}_t} \right] \\ & \times (P_{t-1} P_{k',t-1} \bar{K}_t - \bar{N}_t) \Bigg\} + W_t^e. \end{aligned} \tag{2.56}$$

where upper bar over a letter denotes its aggregate average value. Because of its direct effect on entrepreneurial net worth, γ_t is referred to as the shock to net worth. For a derivation of the aggregation across entrepreneurs, see the online appendix section 3.

We now turn to the external financing premium for entrepreneurs. The cost to the entrepreneur of internal funds (i.e., his own net worth) is the interest rate R_t which he loses by applying it to capital rather than buying a risk-free domestic asset. The average payment by all entrepreneurs to the bank is the entire object in square brackets in (2.56). So, the term involving μ represents the excess of external funds over the internal cost of funds. As a result, this is one measure of the financing premium in the model. Another is $Z_{t+1} - R_t$, the excess of the interest rate paid by entrepreneurs who are not bankrupt over the risk-free rate. This paper calls this the interest rate spread.

3 Model equations

3.1 Scaling of variables

We adopt the following scaling of variables. The neutral shock to technology is z_t and its growth rate is $\mu_{z,t}$:

$$\frac{z_t}{z_{t-1}} = \mu_{z,t}.$$

The variable Ψ_t is an investment-specific shock to technology and it is convenient to define the following combination of investment-specific and neutral technology:

$$\begin{aligned} z_t^+ &= \Psi_t^{\frac{\alpha}{1-\alpha}} z_t, \\ \mu_{z^+,t} &= \mu_{\Psi,t}^{\frac{\alpha}{1-\alpha}} \mu_{z,t}. \end{aligned} \tag{3.1}$$

Capital, $\bar{K}_t$, and investment, I_t, are scaled by $z_t^+ \Psi_t$. Foreign and domestic inputs into the production of I_t (we denote these by I_t^d and I_t^m, respectively) are scaled by z_t^+. Consumption goods (C_t^m are imported intermediate consumption goods, C_t^d are domestically produced intermediate consumption goods, and C_t are final consumption goods) are scaled by z_t^+. Government expenditure, the real wage and real foreign assets are scaled by z_t^+. Exports (X_t^m are imported intermediate goods for use in producing exports and X_t are final export goods) are scaled by z_t^+. Also, υ_t is the shadow value in utility terms to the household of domestic currency and $\upsilon_t P_t$ is the shadow value of one unit of the homogeneous domestic good. The latter must be multiplied by z_t^+ to induce stationarity. $\tilde{P}_t$ is the within-sector relative price of a good. Thus,

$$
\begin{aligned}
k_{t+1} &= \frac{K_{t+1}}{z_t^+ \Psi_t},\ \bar{k}_{t+1} = \frac{\bar{K}_{t+1}}{z_t^+ \Psi_t},\ i_t^d = \frac{I_t^d}{z_t^+},\ i_t = \frac{I_t}{z_t^+ \Psi_t},\ i_m^t = \frac{I_t^m}{z_t^+},\\
c_t^m &= \frac{C_t^m}{z_t^+},\ c_t^d = \frac{C_t^d}{z_t^+},\ c_t = \frac{C_t}{z_t^+},\ g_t = \frac{G_t}{z_t^+},\ \bar{w}_t = \frac{W_t}{z_t^+ P_t},\ a_t := \frac{S_t A_{t+1}^*}{z_t^+ P_t},\\
x_t^m &= \frac{X_t^m}{z_t^+},\ x_t = \frac{X_t}{z_t^+},\ \psi_{z^+,t} = \upsilon_t P_t z_t^+,\ (y_t =)\tilde{y}_t = \frac{Y_t}{z_t^+},\ \tilde{p}_t = \frac{\tilde{P}_t}{P_t},\\
n_{t+1} &= \frac{\bar{N}_{t+1}}{z_t^+ P_t},\ w^e = \frac{W_t^e}{z_t^+ P_t}.
\end{aligned}
$$

We define the scaled date t price of new installed physical capital for the start of period $t+1$ as $p_{k',t}$ and we define the scaled real rental rate of capital as $\bar{r}_t^k$:

$$
p_{k',t} = \Psi_t P_{k',t},\ \bar{r}_t^k = \Psi_t r_t^k,
$$

where $P_{k',t}$ is in units of the domestic homogeneous good.

The nominal exchange rate is denoted by S_t and its growth rate is s_t:

$$
s_t = \frac{S_t}{S_{t-1}}.
$$

We define the following inflation rates:

$$
\begin{aligned}
\pi_t &= \frac{P_t}{P_{t-1}},\ \pi_t^c = \frac{P_t^c}{P_{t-1}^c},\ \pi_t^* = \frac{P_t^*}{P_{t-1}^*},\\
\pi_t^i &= \frac{P_t^i}{P_{t-1}^i},\ \pi_t^x = \frac{P_t^x}{P_{t-1}^x},\ \pi_t^{m,j} = \frac{P_t^{m,j}}{P_{t-1}^{m,j}},
\end{aligned}
$$

for $j = c, x, i$. Here, P_t is the price of a domestic homogeneous output good, P_t^c is the price of the domestic final consumption goods (i.e., the 'consumer price index'), P_t^* is the price of a foreign homogeneous good, P_t^i is the price of the domestic final investment good and P_t^x is the price (in foreign currency units) of a final export good.

With one exception, we define a lower case price as the corresponding uppercase price divided by the price of the homogeneous good. When the price is denominated in domestic currency units, we divide by the price of the domestic homogeneous good, P_t. When the price is denominated in foreign currency units, we divide by P_t^*, the price of the foreign homogeneous good. The exceptional case has to do with handling of the price of investment goods, P_t^i. This grows at a rate potentially slower than P_t, and we therefore scale it by P_t/Ψ_t. Thus,

$$
\begin{aligned}
p_t^{m,x} &= \frac{P_t^{m,x}}{P_t},\ p_t^{m,c} = \frac{P_t^{m,c}}{P_t},\ p_t^{m,i} = \frac{P_t^{m,i}}{P_t},\\
p_t^x &= \frac{P_t^x}{P_t^*},\ p_t^c = \frac{P_t^c}{P_t},\ p_t^i = \frac{\Psi_t P_t^i}{P_t}. \qquad (3.2)
\end{aligned}
$$

Here, m, j means the price of an imported good which is subsequently used in the production of exports in the case $j = x$, in the production of the final consumption good in the case of $j = c$ and in the production of final investment good in the case of $j = i$. When there is just a single superscript the underlying good is a final good, with $j = x, c, i$ corresponding to exports, consumption and investment, respectively.

3.2 Functional forms

We adopt the following functional form for capital utilization, a:

$$a(u) = 0.5\sigma_b\sigma_a u^2 + \sigma_b(1-\sigma_a)u + \sigma_b((\sigma_a/2)-1), \tag{3.3}$$

where σ_a and σ_b are the parameters of this function.

The functional form for investment adjustment costs as well as its derivatives are

$$\begin{aligned}\tilde{S}(x) &= \frac{1}{2}\left\{\exp\left[\sqrt{\tilde{S}''}(x-\mu_{z^+}\mu_\Psi)\right] + \exp\left[-\sqrt{\tilde{S}''}(x-\mu_{z^+}\mu_\Psi)\right] - 2\right\}\\ &= 0,\ x = \mu_{z^+}\mu_\Psi,\end{aligned} \tag{3.4}$$

$$\begin{aligned}\tilde{S}'(x) &= \frac{1}{2}\sqrt{\tilde{S}''}\left\{\exp\left[\sqrt{\tilde{S}''}(x-\mu_{z^+}\mu_\Psi)\right] - \exp\left[-\sqrt{\tilde{S}''}(x-\mu_{z^+}\mu_\Psi)\right]\right\}\\ &= 0,\ x = \mu_{z^+}\mu_\Psi,\end{aligned} \tag{3.5}$$

$$\begin{aligned}\tilde{S}''(x) &= \frac{1}{2}\tilde{S}''\left\{\exp\left[\sqrt{\tilde{S}''}(x-\mu_{z^+}\mu_\Psi)\right] + \exp\left[-\sqrt{\tilde{S}''}(x-\mu_{z^+}\mu_\Psi)\right]\right\}\\ &= \tilde{S}'',\ x = \mu_{z^+}\mu_\Psi.\end{aligned}$$

3.3 Baseline model

3.3.1 First order conditions for domestic homogeneous goods price setting

Substituting (2.7) into (2.6) and rearranging,

$$E_t\sum_{j=0}^{\infty}\beta^j\upsilon_{t+j}P_{t+j}Y_{t+j}\left\{\left(\frac{P_{i,t+j}}{P_{t+j}}\right)^{1-\frac{\lambda_d}{\lambda_d-1}} - mc_{t+j}\left(\frac{P_{i,t+j}}{P_{t+j}}\right)^{\frac{-\lambda_d}{\lambda_d-1}}\right\},$$

or,

$$E_t\sum_{j=0}^{\infty}\beta^j\upsilon_{t+j}P_{t+j}Y_{t+j}\left\{(X_{t,j}\tilde{p}_t)^{1-\frac{\lambda_d}{\lambda_d-1}} - mc_{t+j}(X_{t,j}\tilde{p}_t)^{\frac{-\lambda_d}{\lambda_d-1}}\right\},$$

where

$$\frac{P_{i,t+j}}{P_{t+j}} = X_{t,j}\tilde{p}_t,\ X_{t,j} := \begin{cases}\frac{\tilde{\pi}_{d,t+j}\cdots\tilde{\pi}_{d,t+1}}{\pi_{t+j}\cdots\pi_{t+1}}, & j>0\\ 1, & j=0\end{cases}.$$

The i-th firm maximizes profits by choice of the within-sector relative price $\tilde{p}_t$. The fact that this variable does not have an index i reflects that all firms that have the opportunity to reoptimize in period t solve the same problem, and hence have the same solution. Differentiating its profit function, multiplying the result by $\tilde{p}_t^{\frac{\lambda_d}{\lambda_d-1}+1}$, rearranging, and scaling, yields

$$E_t\sum_{j=0}^{\infty}(\beta\xi_d)^j A_{t+j}[\tilde{p}_t X_{t,j} - \lambda_d mc_{t+j}] = 0,$$

where A_{t+j} is exogenous from the point of view of the firm:

$$A_{t+j} = \psi_{z^+,t+j}\tilde{y}_{t+j}X_{t,j}.$$

After rearranging the optimizing intermediate good firm's first order condition for prices, yields

$$\tilde{p}_t^d = \frac{E_t\sum_{j=0}^{\infty}(\beta\xi_d)^j A_{t+j}\lambda_d mc_{t+j}}{E_t\sum_{j=0}^{\infty}(\beta\xi_d)^j A_{t+j}X_{t,j}} = \frac{K_t^d}{F_t^d},$$

where

$$K_t^d := E_t \sum_{j=0}^{\infty} (\beta\xi_d)^j A_{t+j} \lambda_d mc_{t+j}$$

$$F_t^d := E_t \sum_{j=0}^{\infty} (\beta\xi_d)^j A_{t+j} X_{t,j}.$$

These objects have the following convenient recursive representations

$$E_t \left[\psi_{z^+,t} \tilde{y}_t + \left(\frac{\tilde{\pi}_{d,t+1}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_d}} \beta\xi_d F_{t+1}^d - F_t^d \right] = 0$$

$$E_t \left[\lambda_d \psi_{z^+,t} \tilde{y}_t mc_t + \beta\xi_d \left(\frac{\tilde{\pi}_{d,t+1}}{\pi_{t+1}} \right)^{\frac{\lambda_d}{1-\lambda_d}} K_{t+1}^d - K_t^d \right] = 0.$$

Turning to the aggregate price index:

$$\begin{aligned} P_t &= \left[\int_0^1 P_{it}^{\frac{1}{1-\lambda_d}} di \right]^{1-\lambda_d} \\ &= \left[(1-\xi_p) \tilde{P}_t^{\frac{1}{1-\lambda_d}} + \xi_p (\tilde{\pi}_{d,t} P_{t-1})^{\frac{1}{1-\lambda_d}} \right]^{1-\lambda_d} \end{aligned} \tag{3.6}$$

After dividing by P_t and rearranging

$$\frac{1 - \xi_d \left(\frac{\tilde{\pi}_{d,t}}{\pi_t} \right)^{\frac{1}{1-\lambda_d}}}{1-\xi_d} = (\tilde{p}_t^d)^{\frac{1}{1-\lambda_d}}. \tag{3.7}$$

In sum, the equilibrium conditions associated with price setting for producers of the domestic homogeneous good are[18]

$$E_t \left[\psi_{z^+,t} y_t + \left(\frac{\tilde{\pi}_{d,t+1}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_d}} \beta\xi_d F_{t+1}^d - F_t^d \right] = 0 \tag{3.8}$$

$$E_t \left[\lambda_d \psi_{z^+,t} y_t mc_t + \beta\xi_d \left(\frac{\tilde{\pi}_{d,t+1}}{\pi_{t+1}} \right)^{\frac{\lambda_d}{1-\lambda_d}} K_{t+1}^d - K_t^d \right] = 0 \tag{3.9}$$

$$\mathring{p}_t = \left[(1-\xi_d) \left(\frac{1 - \xi_d \left(\frac{\tilde{\pi}_{d,t}}{\pi_t} \right)^{\frac{1}{1-\lambda_d}}}{1-\xi_d} \right)^{\lambda_d} + \xi_d \left(\frac{\tilde{\pi}_{d,t}}{\pi_t} \mathring{p}_{t-1} \right)^{\frac{\lambda_d}{1-\lambda_d}} \right]^{\frac{1-\lambda_d}{\lambda_d}} \tag{3.10}$$

$$\left[\frac{1 - \xi_d \left(\frac{\tilde{\pi}_{d,t}}{\pi_t} \right)^{\frac{1}{1-\lambda_d}}}{1-\xi_d} \right]^{1-\lambda_d} = \frac{K_t^d}{F_t^d} \tag{3.11}$$

$$\tilde{\pi}_{d,t} := (\pi_{t-1})^{\kappa_d} (\bar{\pi}_t^c)^{1-\kappa_d-\varkappa_d} (\breve{\pi})^{\varkappa_d} \tag{3.12}$$

[18] After linearizing about the steady state and setting $\varkappa_d = 0$,

$$\begin{aligned} \hat{\pi} - \hat{\bar{\pi}}_t^c = &\frac{\beta}{1+\kappa_d\beta} E_t(\hat{\pi}_{t+1} - \hat{\bar{\pi}}_{t+1}^c) + \frac{\kappa_d}{1+\kappa_d\beta}(\hat{\pi}_{t-1} - \hat{\bar{\pi}}_t^c) \\ &- \frac{\kappa_d\beta(1-\rho_\pi)}{1+\kappa_d\beta} \hat{\bar{\pi}}_t^c \\ &+ \frac{1}{1+\kappa_d\beta} \frac{(1-\beta\xi_d)(1-\xi_d)}{\xi_d} \widehat{mc}_t, \end{aligned}$$

where a hat indicates log-deviation from steady state.

3.3.2 Export demand

Scaling (2.17), yields

$$x_t = (p_t^x)^{-\eta_f} y_t^* \tag{3.13}$$

3.3.3 FOCs for export goods price setting

$$E_t\left[\psi_{z^+,t} q_t p_t^c p_t^x x_t + \left(\frac{\tilde{\pi}_{t+1}^x}{\pi_{t+1}^x}\right)^{\frac{1}{1-\lambda_x}} \beta\xi_x F_{x,t+1} - F_{x,t}\right] = 0 \tag{3.14}$$

$$E_t\left[\lambda_x \psi_{z^+,t} q_t p_t^c p_t^x x_t mc_t^x + \beta\xi_x \left(\frac{\tilde{\pi}_{t+1}^x}{\pi_{t+1}^x}\right)^{\frac{\lambda_x}{1-\lambda_x}} K_{x,t+1} - K_{x,t}\right] = 0 \tag{3.15}$$

$$\mathring{p}_t^x = \left[(1-\xi_x)\left(\frac{1-\xi_x\left(\frac{\tilde{\pi}_t^x}{\pi_t^x}\right)\frac{1}{1-\lambda_x}}{1-\xi_x}\right)^{\lambda_x} + \xi_x\left(\frac{\tilde{\pi}_t^x}{\pi_t^x}\mathring{p}_{t-1}^x\right)^{\frac{\lambda_x}{1-\lambda_x}}\right]^{\frac{1-\lambda_x}{\lambda_x}} \tag{3.16}$$

$$\left[\frac{1-\xi_x\left(\frac{\tilde{\pi}_t^x}{\pi_t^x}\right)^{\frac{1}{1-\lambda_x}}}{1-\xi_x}\right]^{1-\lambda_x} = \frac{K_{x,t}}{F_{x,t}} \tag{3.17}$$

When linearized around steady state and $\varkappa_{m,j} = 0$, eq. (3.14)-(3.17) reduce to

$$\begin{aligned}\hat{\pi}_t^x =& \frac{\beta}{1+\kappa_x\beta} E_t\hat{\pi}_{t+1}^x + \frac{\kappa_x}{1+\kappa_x\beta}\hat{\pi}_{t-1}^x \\ &+ \frac{1}{1+\kappa_x\beta}\frac{(1-\beta\xi_x)(1-\xi_x)}{\xi_x}\widehat{mc}_t^x\end{aligned}$$

where a hat over a variable indicates log-deviation from steady state.

3.3.4 Demand for domestic inputs in export production

Integrating (2.24),

$$\begin{aligned}\int_0^1 X_{i,t}^d di &= \left(\frac{\lambda}{\tau_t^x R_t^x P_t}\right)^{\eta_x} (1-\omega_x)\int_0^1 X_{i,t} di \\ &= \left(\frac{\lambda}{\tau_t^x R_t^x P_t}\right)^{\eta_x} (1-\omega_x) X_t \frac{\int_0^1 (P_{i,t}^x)^{\frac{-\lambda_x}{\lambda_x-1}} di}{(P_t^x)^{\frac{-\lambda_x}{\lambda_x-1}}}\end{aligned} \tag{3.18}$$

Define $\mathring{P}_t^x$, a linear homogeneous function of $P_{i,t}^x$:

$$\mathring{P}_t^x = \left[\int_0^1 (P_{i,t}^x)^{\frac{-\lambda_x}{\lambda_x-1}} di\right]^{\frac{\lambda_x-1}{-\lambda_x}}.$$

Then

$$\left(\mathring{P}_t^x\right)^{\frac{-\lambda_x}{\lambda_x-1}} = \int_0^1 (P_{i,t}^x)^{\frac{-\lambda_x}{\lambda_x-1}} di$$

and

$$\int_0^1 X_{i,t}^d di = \left(\frac{\lambda}{\tau_t^x R_t^x P_t}\right)^{\eta_x} (1-\omega_x) X_t (\mathring{p}_t^x)^{\frac{-\lambda_x}{\lambda_x-1}} \tag{3.19}$$

where

$$\mathring{p}_t^x := \frac{\mathring{P}_t^x}{P_t^x}$$

and the law of motion of $\mathring{p}_t^x$ is given in (3.16).

We now simplify (3.19). Rewriting the second equality in (2.20), yields

$$\frac{\lambda}{P_t \tau_t^x R_t^x} = \frac{S_t P_t^x}{P_t q_t p_t^c p_t^x} \left[\omega_x (p_t^{m,x})^{1-\eta_x} + (1-\omega_x)\right]^{\frac{1}{1-\eta_x}}$$

or

$$\frac{\lambda}{P_t \tau_t^x R_t^x} = \frac{S_t P_t^x}{P_t \frac{S_t P_t^*}{P_t^c} \frac{P_t^c}{P_t} \frac{P_t^x}{P_t^*}} \left[\omega_x (p_t^{m,x})^{1-\eta_x} + (1-\omega_x)\right]^{\frac{1}{1-\eta_x}}$$

or

$$\frac{\lambda}{P_t \tau_t^x R_t^x} = \left[\omega_x (p_t^{m,x})^{1-\eta_x} + (1-\omega_x)\right]^{\frac{1}{1-\eta_x}}.$$

Substituting into (3.19),

$$X_t^d = \int_0^1 X_{i,t}^d di = \left[\omega_x (p_t^{m,x})^{1-\eta_x} + (1-\omega_x)\right]^{\frac{\eta_x}{1-\eta_x}} (1-\omega_x) (\mathring{p}_t^x)^{\frac{-\lambda_x}{\lambda_x - 1}} (p_t^x)^{-\eta_x} Y_t^*$$

3.3.5 Demand for imported inputs in export production

Scaling (2.26), yields

$$x_t^m = \omega_x \left(\frac{\left[\omega_x (p_t^{m,x})^{1-\eta_x} + (1-\omega_x)\right]^{\frac{1}{1-\eta_x}}}{p_t^{m,x}} \right)^{\eta_x} (\mathring{p}_t^x)^{\frac{-\lambda_x}{\lambda_x - 1}} (p_t^x)^{-\eta_f} y_t^* \tag{3.20}$$

3.3.6 Value of imports of the intermediate consumption goods producers

It is of interest to have a measure of the value of imports of the intermediate consumption good producers:

$$S_t P_t^* R_t^{\nu,*} \int_0^1 C_{i,t}^m di.$$

In order to relate this to C_t^m, substitute the demand curve into the previous expression:

$$\begin{aligned} S_t P_t^* R_t^{\nu,*} \int_0^1 C_t^m \left(\frac{P_t^{m,c}}{P_{i,t}^{m,c}} \right)^{\frac{\lambda_{m,c}}{\lambda_{m,c}-1}} di &= S_t P_t^* R_t^{\nu,*} C_t^m (P_t^{m,c})^{\frac{\lambda_{m,c}}{\lambda_{m,c}-1}} \int_0^1 (P_{i,t}^{m,c})^{\frac{-\lambda_{m,c}}{\lambda_{m,c}-1}} di \\ &= S_t P_t^* R_t^{\nu,*} C_t^m \left(\frac{\mathring{P}_t^{m,c}}{P_t^{m,c}} \right)^{\frac{\lambda_{m,c}}{1-\lambda_{m,c}}}, \end{aligned}$$

where

$$\mathring{P}_t^{m,c} = \left[\int_0^1 (P_{i,t}^{m,c})^{\frac{\lambda_{m,c}}{1-\lambda_{m,c}}} \right]^{\frac{1-\lambda_{m,c}}{\lambda_{m,c}}}.$$

Thus the total value of imports accounted for by the consumption sector is

$$S_t P_t^* R_t^{\nu,*} C_t^m (\mathring{p}_t^{m,c})^{\frac{\lambda_{m,c}}{1-\lambda_{m,c}}} \tag{3.21}$$

where

$$\mathring{p}_t^{m,c} = \frac{\mathring{P}_t^{m,c}}{P_t^{m,c}}.$$

The derivation for the value of imports used by the investment and export production sectors are analogous.

3.3.7 Marginal costs of importers

Real marginal cost is

$$
\begin{aligned}
mc_t^{m,j} &= \tau_t^{m,j} \frac{S_t P_t^*}{P_t^{m,j}} R_t^{\nu,*} = \tau_t^{m,j} \frac{S_t P_t^* P_t^c P_t}{P_t^c P_t^{m,j} P_t} R_t^{\nu,*} \\
&= \tau_t^{m,j} \frac{q_t p_t^c}{p_t^{m,j}} R_t^{\nu,*} \qquad (3.22)
\end{aligned}
$$

for $j = c, i, x$.

3.3.8 FOCs for import goods price setting

$$
E_t \left[\psi_{z^+,t} p_t^{m,j} \Xi_t^j + \left(\frac{\tilde{\pi}_{t+1}^{m,j}}{\pi_{t+1}^{m,j}} \right)^{\frac{1}{1-\lambda_{m,j}}} \beta \xi_{m,j} F_{m,j,t+1} - F_{m,j,t} \right] = 0 \qquad (3.23)
$$

$$
E_t \left[\lambda_{m,j} \psi_{z^+,t} p_t^{m,j} mc_t^{m,j} \Xi_t^j + \beta \xi_{m,j} \left(\frac{\tilde{\pi}_{t+1}^{m,j}}{\pi_{t+1}^{m,j}} \right)^{\frac{\lambda_{m,j}}{1-\lambda_{m,j}}} K_{m,j,t+1} - K_{m,j,t} \right] = 0 \qquad (3.24)
$$

$$
\mathring{p}_t^{m,j} = \left[(1-\xi_{m,j}) \left(\frac{1 - \xi_{m,j} \left(\frac{\tilde{\pi}_t^{m,j}}{\pi_t^{m,j}} \right)^{\frac{1}{1-\lambda_{m,j}}}}{1-\xi_{m,j}} \right)^{\lambda_{m,j}} + \xi_{m,j} \left(\frac{\tilde{\pi}_t^{m,j}}{\pi_t^{m,j}} \mathring{p}_{t-1}^{m,j} \right)^{\frac{\lambda_{m,j}}{1-\lambda_{m,j}}} \right]^{\frac{1-\lambda_{m,j}}{\lambda_{m,j}}} \qquad (3.25)
$$

$$
\left[\frac{1 - \xi_{m,j} \left(\frac{\tilde{\pi}_t^{m,j}}{\pi_t^{m,j}} \right)^{\frac{1}{1-\lambda_{m,j}}}}{1-\xi_{m,j}} \right]^{1-\lambda_{m,j}} = \frac{K_{m,j,t}}{F_{m,j,t}} \qquad (3.26)
$$

for $j = c, t, x$,[19] and where

$$
\Xi_t^j = \begin{cases} c_t^m & j = c \\ x_t^m & j = x \\ i_t^m & j = i \end{cases} .
$$

3.3.9 Wage setting conditions in baseline model

Substituting (2.37) into the objective function, (2.36),

$$
\begin{aligned}
& E_t^j \sum_0^\infty (\beta \xi_w)^i \Bigg[-\zeta_{t+i}^h A_L \frac{\left(\left(\frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\sigma_L}}{1+\sigma_L} \\
& + \upsilon_{t+i} \tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1} \left(\frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau^y}{1+\tau^w} \Bigg]
\end{aligned}
$$

[19] When linearized around steady state and $\varkappa_{m,j} = 0$,

$$
\begin{aligned}
\hat{\pi}_t^{m,j} - \hat{\bar{\pi}}_t^c =& \frac{\beta}{1+\kappa_{m,j}\beta} E_t \left(\hat{\pi}_{t+1}^{m,j} - \hat{\bar{\pi}}_{t+1}^c \right) + \frac{\kappa_{m,j}}{1+\kappa_{m,j}\beta} \left(\hat{\pi}_{t-1}^{m,j} - \hat{\bar{\pi}}_t^c \right) \\
& - \frac{\kappa_{m,j}\beta(1-\rho_\pi)}{1+\kappa_{m,j}\beta} \hat{\bar{\pi}}_t^c \\
& + \frac{1}{1+\kappa_{m,j}\beta} \frac{(1-\beta\xi_{m,j})(1-\xi_{m,j})}{\xi_{m,j}} \widehat{mc}_t^{m,j}.
\end{aligned}
$$

Given the rescaled variables,

$$\begin{aligned}\frac{\tilde{W}_t \tilde{\pi}_{w,t+i}\cdots\tilde{\pi}_{w,t+1}}{W_{t+i}} &= \frac{\tilde{W}_t \tilde{\pi}_{w,t+1}\cdots\tilde{\pi}_{w,t+1}}{\bar{w}_{t+i} z^+_{t+i} P_{t+i}} = \frac{\tilde{W}_t}{\bar{w}_{t+i} z^+_t P_t} X_{t,i} \\ &= \frac{W_t \left(\tilde{W}_t / W_t\right)}{\bar{w}_{t+i} z^+_t P_t} X_{t,i} = \frac{\bar{w}_t \left(\tilde{W}_t / W_t\right)}{\bar{w}_{t+i}} X_{t,i} = \frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i},\end{aligned}$$

where

$$X_{t,i} = \begin{cases} \frac{\tilde{\pi}_{w,t+i}\cdots\tilde{\pi}_{w,t+1}}{\pi_{t+i}\pi_{t+i-1}\cdots\pi_{t+1}\mu_{z^+,t+i}\cdots\mu_{z^+,t+1}}, & i > 0 \\ 1, & i = 0 \end{cases}.$$

It is interesting to investigate the value of $X_{t,i}$ in steady state, as $i \to \infty$. Thus,

$$X_{t,i} = \frac{\left(\pi^c_t \cdots \pi^c_{t+i-1}\right)^{\kappa_w} \left(\bar{\pi}^c_{t+1} \cdots \bar{\pi}^c_{t+i}\right)^{1-\kappa_w-\varkappa_w} \left(\breve{\pi}^i\right)^{\varkappa_w} \left(\mu^i_{z^+}\right)^{\vartheta_w}}{\pi_{t+i}\pi_{t+i-1}\cdots\pi_{t+1}\mu_{z^+,t+i}\cdots\mu_{z^+,t+1}}$$

In steady state,

$$\begin{aligned}X_{t,i} &= \frac{\left(\bar{\pi}^i\right)^{\kappa_w} \left(\bar{\pi}^i\right)^{1-\kappa_w-\varkappa_w} \left(\breve{\pi}^i\right)^{\varkappa_w} \left(\mu^i_{z^+}\right)^{\vartheta_w}}{\bar{\pi}^i \mu^i_{z^+}} \\ &= \left(\frac{\breve{\pi}^i}{\bar{\pi}^i}\right)^{\varkappa_w} \left(\mu^i_{z^+}\right)^{\vartheta_w - 1} \\ &\to 0\end{aligned}$$

in the no-indexing case, when $\breve{\pi} = 1$, $\varkappa_w = 1$ and $\vartheta_w = 0$.

Simplifying using the scaling notation,

$$\begin{aligned}E^j_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \Bigg[&-\zeta^h_{t+i} A_L \frac{\left(\left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}\right)^{1+\sigma_L}}{1+\sigma_L} \\ &+ \upsilon_{t+i} W_{t+i} \frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau^y}{1+\tau^w} \Bigg]\end{aligned}$$

or

$$\begin{aligned}E^j_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \Bigg[&-\zeta^h_{t+i} A_L \frac{\left(\left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}\right)^{1+\sigma_L}}{1+\sigma_L} \\ &+ \psi_{z^+,t+i} w_t \bar{w}_t X_{t,i} \left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau^y}{1+\tau^w} \Bigg]\end{aligned}$$

or

$$\begin{aligned}E^j_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \Bigg[&-\zeta^h_{t+i} A_L \frac{\left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}\right)^{1+\sigma_L}}{1+\sigma_L} w_t^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} \\ &+ \psi_{z^+,t+i} w_t^{1+\frac{\lambda_w}{1-\lambda_w}} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau^y}{1+\tau^w} \Bigg]\end{aligned}$$

Differentiating with respect to w_t and solving for the wage rate [skipped some math]

$$w_t^{\frac{1-\lambda_w(1+\sigma_L)}{1-\lambda_w}} = \frac{E_t^j \sum_{i=0}^{\infty}(\beta\xi_w)^i \zeta_{t+i}^h A_L \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}\right)^{1+\sigma_L}}{E_t^j \sum_{i=0}^{\infty}(\beta\xi_w)^i \frac{\psi_{z^+,t+i}}{\lambda_w} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau^y}{1+\tau^w}}$$
$$= \frac{A_L K_{w,t}}{\bar{w}_t F_{w,t}}$$

where

$$K_{w,t} := E_t^j \sum_{i=0}^{\infty}(\beta\xi_w)^i \zeta_{t+i}^h \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}\right)^{1+\sigma_L}$$
$$F_{w,t} := E_t^j \sum_{i=0}^{\infty}(\beta\xi_w)^i \frac{\psi_{z^+,t+i}}{\lambda_w} X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau^y}{1+\tau^w}.$$

Thus, the wage set by reoptimizing households is

$$w_t = \left[\frac{A_L K_{w,t}}{\bar{w}_t F_{w,t}}\right]^{\frac{1-\lambda_w}{1-\lambda_w(1+\sigma_L)}}.$$

We now express $K_{w,t}$ and $F_{w,t}$ in recursive form [after some skipped math]:

$$K_{w,t} = \zeta_t^h H_t^{1+\sigma_L} + \beta\xi_w E_t \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}}\right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} K_{w,t+1}$$

where

$$\pi_{w,t+1} = \frac{W_{t+1}}{W_t} = \frac{\bar{w}_{t+1} z_{t+1}^+ P_{t+1}}{\bar{w}_t z_t^+ P_t} = \frac{\bar{w}_{t+1}\mu_{z^+,t+1}\pi_{t+1}}{\bar{w}_t} \tag{3.27}$$

Also [after some skipped math],

$$F_{w,t} = \frac{\psi_{z^+,t}}{\lambda_w} H_t \frac{1-\tau^y}{1+\tau^w} + \beta\xi_w E_t \left(\frac{\bar{w}_{t+1}}{\bar{w}_t}\right)\left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} F_{w,t+1}.$$

The second restriction on w_t is obtained using the relation between the aggregate wage rate and the wage rates of individual households:

$$W_t = \left[(1-\xi_w)\left(\tilde{W}_t\right)^{\frac{1}{1-\lambda_w}} + \xi_w(\tilde{\pi}_{w,t} W_{t-1})^{\frac{1}{1-\lambda_w}}\right]^{1-\lambda_w}.$$

Dividing both sides by W_t and rearranging,

$$w_t = \left[\frac{1-\xi_w\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right]^{1-\lambda_w}$$

Substituting out for w_t from the household's FOC for wage optimization,

$$\frac{1}{A_L}\left[\frac{1-\xi_w\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right]^{1-\lambda_w(1+\sigma_L)} \bar{w}_t F_{w,t} = K_{w,t}.$$

We now derive the relationship between aggregate homogeneous hours worked, H_t, and aggregate household hours,

$$h_t := \int_0^1 h_{j,t} dj.$$

Substituting the demand for $h_{j,t}$ into the latter expression,

$$\begin{aligned} h_t &= \int_0^1 \left(\frac{W_{j,t}}{W_t} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_t dj \\ &= \frac{H_t}{(W_t)^{\frac{\lambda_w}{1-\lambda_w}}} \int_0^1 (W_{j,t})^{\frac{\lambda_w}{1-\lambda_w}} dj \\ &= \mathring{w}_t^{\frac{\lambda_w}{1-\lambda_w}} H_t, \end{aligned} \tag{3.28}$$

where

$$\mathring{w}_t = \frac{\mathring{W}_t}{W_t},\ \mathring{W}_t = \left[\int_0^1 (W_{j,t})^{\frac{\lambda_w}{1-\lambda_w}} dj \right]^{\frac{1-\lambda_w}{\lambda_w}}$$

and

$$\mathring{W}_t = \left[(1-\xi_w) \left(\tilde{W}_t \right)^{\frac{\lambda_w}{1-\lambda_w}} + \xi_w \left(\tilde{\pi}_{w,t} \mathring{W}_{t-1} \right)^{\frac{\lambda_w}{1-\lambda_w}} \right]^{\frac{1-\lambda_w}{\lambda_w}},$$

so that

$$\begin{aligned} \mathring{w}_t &= \left[(1-\xi_w)(w_t)^{\frac{\lambda_w}{1-\lambda_w}} + \xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \mathring{w}_{t-1} \right)^{\frac{\lambda_w}{1-\lambda_w}} \right]^{\frac{1-\lambda_w}{\lambda_w}} \\ &= \left[(1-\xi_w) \left(\frac{1-\xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w} \right)^{\lambda_w} + \xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \mathring{w}_{t-1} \right)^{\frac{\lambda_w}{1-\lambda_w}} \right]^{\frac{1-\lambda_w}{\lambda_w}}. \end{aligned} \tag{3.29}$$

In addition to (3.29), we have the following equilibrium conditions associated with sticky wages:

$$F_{w,t} = \frac{\psi_{z^+,t}}{\lambda_w} \mathring{w}_t^{\frac{-\lambda_w}{1-\lambda_w}} h_t \frac{1-\tau^y}{1+\tau^w} + \beta \xi_w E_t \left(\frac{\bar{w}_{t+1}}{\bar{w}_t} \right) \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}} \right)^{1+\frac{\lambda_w}{1-\lambda_w}} F_{w,t+1} \tag{3.30}$$

$$K_{w,t} = \zeta_t^h \left(\mathring{w}_t^{\frac{-\lambda_w}{1-\lambda_w}} h_t \right)^{1+\sigma_L} + \beta \xi_w E_t \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} K_{w,t+1} \tag{3.31}$$

$$\frac{1}{A_L} \left[\frac{1-\xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w} \right]^{1-\lambda_w(1+\sigma_L)} \bar{w}_t F_{w,t} = K_{w,t}. \tag{3.32}$$

3.3.10 Scaling law of motion of capital

Using (2.38), the law of motion of capital in scaled terms is

$$\bar{k}_{t+1} = \frac{1-\delta}{\mu_{z^+,t}\mu_{\Psi,t}} \bar{k}_t + \Upsilon_t \left(1 - \tilde{S} \left(\frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}} \right) \right) i_t. \tag{3.33}$$

3.3.11 Output and aggregate factors of production

Below we derive a relationship between total output of the domestic homogeneous good, Y_t, and aggregate factors of production.

Consider the unweighted average of the intermediate goods:

$$\begin{aligned} Y_t^{sum} &= \int_0^1 Y_{i,t} di \\ &= \int_0^1 \left[(z_t H_{i,t})^{1-\alpha} \epsilon_t K_{i,t}^{\alpha} - z_t^+ \phi \right] di \\ &= \int_0^1 \left[z_t^{1-\alpha} \epsilon_t \left(\frac{K_{i,t}}{H_{i,t}} \right)^{\alpha} H_{i,t} - z_t^+ \phi \right] di \\ &= z_t^{1-\alpha} \epsilon_t \left(\frac{K_t}{H_t} \right)^{\alpha} \int_0^1 H_{i,t} di - z_t^+ \phi, \end{aligned}$$

where K_t is the economy-wide average stock of capital services and H_t is the economy-wide average of homogeneous labor. The last expression exploits the fact that all intermediate good firms confront the same factor prices, and so they adopt the same capital services to homogeneous labor ratio. This follows from cost minimization, and holds for all firms, regardless whether or not they have an opportunity to reoptimize. Then,

$$Y_t^{sum} = z_t^{1-\alpha} \epsilon_t K_t^{\alpha} H_t^{1-\alpha} - z_t^+ \phi.$$

Recall that the demand for $Y_{j,t}$ is

$$\left(\frac{P_t}{P_{i,t}} \right)^{\frac{\lambda_d}{\lambda_d - 1}} = \frac{Y_{i,t}}{Y_t},$$

so that

$$\mathring{Y}_t := \int_0^1 Y_{i,t} di = \int_0^1 Y_t \left(\frac{P_t}{P_{i,t}} \right)^{\frac{\lambda_d}{\lambda_d - 1}} di = Y_t P_t^{\frac{\lambda_d}{\lambda_d - 1}} \left(\mathring{P}_t \right)^{\frac{\lambda_d}{1 - \lambda_d}},$$

where

$$\mathring{P}_t = \left[\int_0^1 P_{i,t}^{\frac{\lambda_d}{1-\lambda_d}} di \right]^{\frac{1-\lambda_d}{\lambda_d}}. \tag{3.34}$$

Dividing by P_t,

$$\mathring{p}_t = \left[\int_0^1 \left(\frac{P_{it}}{P_t} \right)^{\frac{\lambda_d}{1-\lambda_d}} di \right]^{\frac{1-\lambda_d}{\lambda_d}},$$

or,

$$\mathring{p}_t = \left[(1 - \xi_p) \left(\frac{1 - \xi_p \left(\frac{\tilde{\pi}_{d,t}}{\pi_t} \right)^{\frac{1}{1-\lambda_d}}}{1 - \xi_p} \right)^{\lambda_d} + \xi_p \left(\frac{\tilde{\pi}_{d,t}}{\pi_t} \mathring{p}_{t-1} \right)^{\frac{\lambda_d}{1-\lambda_d}} \right]^{\frac{1-\lambda_d}{\lambda_d}}. \tag{3.35}$$

The preceding implies

$$Y_t = (\mathring{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} \mathring{Y}_t = (\mathring{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} \left[z_t^{1-\alpha} \epsilon_t K_t^{\alpha} H_t^{1-\alpha} - z_t^+ \phi \right],$$

or, after scaling by z_t^+,

$$y_t = (\mathring{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} \left[\epsilon_t \left(\frac{1}{\mu_{\Psi,t}\mu_{z^+,t}} k_t \right)^{\alpha} H_t^{1-\alpha} - \phi \right],$$

where

$$k_t = \bar{k}_t u_t. \tag{3.36}$$

Plugging H_t from (3.28),

$$y_t = (\mathring{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} \left[\epsilon_t \left(\frac{1}{\mu_{\Psi,t}\mu_{z^+,t}} k_t \right)^{\alpha} \left(\mathring{w}_t^{\frac{\lambda_w}{1-\lambda_w}} h_t \right)^{1-\alpha} - \phi \right].$$

3.3.12 Restrictions across inflation rates

We now consider the restrictions across inflation rates implied by the relative price formulas. In terms of the expressions in (3.2), there are the restrictions implied by $P_t^{m,j}/p_{t-1}^{m,j}$, $j = x, c, i$, and p_t^x. The restrictions implied by the other two relative prices in (3.2), p_t^i and p_t^c, have already been used in (2.16) and (3.33), respectively. Finally, we also use the restriction across inflation rates implied by q_t/q_{t-1} and (2.23). Thus,

$$\frac{p_t^{m,x}}{p_{t-1}^{m,x}} = \frac{\pi_t^{m,x}}{\pi_t} \tag{3.37}$$

$$\frac{p_t^{m,c}}{p_{t-1}^{m,c}} = \frac{\pi_t^{m,c}}{\pi_t} \tag{3.38}$$

$$\frac{p_t^{m,i}}{p_{t-1}^{m,i}} = \frac{\pi_t^{m,i}}{\pi_t} \tag{3.39}$$

$$\frac{p_t^{x}}{p_{t-1}^{x}} = \frac{\pi_t^{x}}{\pi_t^*} \tag{3.40}$$

$$\frac{q_t}{q_{t-1}} = \frac{s_t \pi_t^*}{\pi_t^c}. \tag{3.41}$$

3.3.13 Endogenous variables of the baseline model

In above we derived the following 70 equations:

(2.3), (2.4), (2.5), (3.8), (3.9), (3.10), (3.11), (3.12), (3.3), (2.10), (2.11), (2.12), (2.15), (2.16), (2.14), (3.13), (2.21), (2.20), (2.27), (3.14), (3.15), (3.16), (3.17), (3.20), (2.29), (3.23), (3.24), (3.25), (3.26), (2.32), (3.22), (3.4), (3.5), (3.33), (2.39), (2.41), (2.42), (2.43), (2.44), (2.45), (2.47), (3.30), (3.31), (3.32), (3.29), (2.35), (3.27), (3.28), (3.36), (2.49), (2.51), (2.50), (3.37), (3.38), (3.39), (3.40), (3.41), (2.48),

which can be used to solve for the following 70 unknowns:

$\bar{r}_t^k$, $\bar{w}_t$, $R_t^{\nu,*}$, R_t^f, R_t^x, R_t, mc_t, mc_t^x, $mc_t^{m,c}$, $mc_t^{m,i}$, $mc_t^{m,x}$, π_t, π_t^x, π_t^c, π_t^i, $\pi_t^{m,c}$, $\pi_t^{m,i}$, $\pi_t^{m,x}$, p_t^c, p_t^x, p_t^i, $p_t^{m,x}$, $p_t^{m,c}$, $p_t^{m,i}$, $p_{k',t}$, k_{t+1}, $\bar{k}_{t+1}$, u_t, h_t, H_t, q_t, i_t, c_t, x_t, a_t, $\psi_{z^+,t}$, y_t, K_t^d, F_t^d, $\tilde{\pi}_{d,t}$, $\mathring{p}_t$,$K_{x,t}$, $F_{x,t}$, $\tilde{\pi}_t^x$, $\mathring{p}_t^x$, $\{K_{m,j,t}$, $F_{m,j,t}$, $\tilde{\pi}_t^{m,j}$, $\mathring{p}_t^{m,j}$; $j = c, i, x\}$, $K_{w,t}$, $F_{w,t}$, $\tilde{\pi}_t^w$, R_t^k, Φ_t, $\tilde{S}_t$, $\tilde{S}_t'$, $a(u_t)$, $\mathring{w}_t$, c_t^m, i_t^m, x_t^m, π_w.

3.4.1 Derivation of optimal contract

As noted in the text, it is supposed that the equilibrium debt contract maximizes entrepreneurial welfare subject to the zero profit condition on banks and the specified required return on household bank liabilities. The date t debt contract specifies a level of debt B_{t+1} and a state $t+1$-contingent rate of interest, Z_{t+1}. We suppose that entrepreneurial welfare corresponds to the entrepreneur's expected wealth at the end of the contract. It is convenient to express welfare as a ratio to the amount the entrepreneur could receive by depositing his net worth in a bank:

$$\frac{E_t \int_{\bar{\omega}_{t+1}}^{\infty} \left[R_{t+1}^k \omega P_t P_{k',t} \bar{K}_{t+1} - Z_{t+1} B_{t+1}\right] dF(\omega;\sigma_t)}{R_t N_{t+1}}$$
$$= \frac{E_t \int_{\bar{\omega}_{t+1}}^{\infty} [\omega - \bar{\omega}_{t+1}] dF(\omega;\sigma_t) R_{t+1}^k P_t P_{k',t} \bar{K}_{t+1}}{R_t N_{t+1}}$$
$$= E_t \left\{ [1 - \Gamma(\bar{\omega}_{t+1};\sigma_t)] \frac{R_{t+1}^k}{R_t} \right\} \varrho_t,$$

after making use of (2.52), (2.53) and

$$1 = \int_0^{\infty} \omega dF(\omega;\sigma_t) = \int_{\bar{\omega}_{t+1}}^{\infty} \omega dF(\omega;\sigma_t) + G(\bar{\omega}_{t+1};\sigma_t).$$

We can equivalently characterize the contract by a state-$t+1$ contingent set of values for $\bar{\omega}_{t+1}$ and a value of ϱ_t. The equilibrium contract is the one involving $\bar{\omega}_{t+1}$ and ϱ_t which maximizes entrepreneurial welfare (relative to $R_t N_{t+1}$) subject to the bank zero profits condition. The Lagrangian representation of this problem is:

$$\max_{\varrho_t, \{\bar{\omega}_{t+1}\}} E_t \left\{ [1 - \Gamma(\bar{\omega}_{t+1};\sigma_t)] \frac{R_{t+1}^k}{R_t} \varrho_t + \lambda_{t+1} \left([\Gamma(\bar{\omega}_{t+1};\sigma_t) - \mu G(\bar{\omega}_{t+1};\sigma_t)] \frac{R_{t+1}^k}{R_t} \varrho_t - \varrho_t + 1 \right) \right\},$$

where λ_{t+1} is the Lagrange multiplier which is defined for each period $t+1$ state of nature. The FOCs for this problem are:

$$E_t \left\{ [1 - \Gamma(\bar{\omega}_{t+1};\sigma_t)] \frac{R_{t+1}^k}{R_t} + \lambda_{t+1} \left([\Gamma(\bar{\omega}_{t+1};\sigma_t) - \mu G(\bar{\omega}_{t+1};\sigma_t)] \frac{R_{t+1}^k}{R_t} - 1 \right) \right\} = 0$$
$$-\Gamma_{\bar{\omega}}(\bar{\omega}_{t+1};\sigma_t) \frac{R_{t+1}^k}{R_t} + \lambda_{t+1} [\Gamma_{\bar{\omega}}(\bar{\omega}_{t+1};\sigma_t) - \mu G_{\bar{\omega}}(\bar{\omega}_{t+1};\sigma_t)] \frac{R_{t+1}^k}{R_t} = 0$$
$$[\Gamma(\bar{\omega}_{t+1};\sigma_t) - \mu G(\bar{\omega}_{t+1};\sigma_t)] \frac{R_{t+1}^k}{R_t} \varrho_t - \varrho_t + 1 = 0,$$

where the absence of λ_{t+1} from the complementary slackness condition reflects that it is assumed that $\lambda_{t+1} > 0$ in each period $t+1$ state of nature. Substituting out for λ_{t+1} from the second equation into the first, the FOCs reduce to

$$E_t \left\{ \begin{array}{c} [1 - \Gamma(\bar{\omega}_{t+1};\sigma_{t+1})] \frac{R_{t+1}^k}{R_t} + \frac{\Gamma_{\bar{\omega}}(\bar{\omega}_{t+1};\sigma_t)}{\Gamma_{\bar{\omega}}(\bar{\omega}_{t+1};\sigma_t) - \mu G_{\bar{\omega}}(\bar{\omega}_{t+1};\sigma_t)} \times \\ \left([\Gamma(\bar{\omega}_{t+1};\sigma_t) - \mu G(\bar{\omega}_{t+1};\sigma_t)] \frac{R_{t+1}^k}{R_t} - 1 \right) \end{array} \right\} = 0 \tag{3.42}$$

$$[\Gamma(\bar{\omega}_{t+1};\sigma_t) - \mu G(\bar{\omega}_{t+1};\sigma_t)] \frac{R_{t+1}^k}{R_t} \varrho_t - \varrho_t + 1 = 0 \tag{3.43}$$

for $t = 0, 1, 2, \ldots, \infty$ and for $t = -1, 0, 1, 2, \ldots$ respectively.

Since N_{t+1} does not appear in the last two equations, ϱ_t and $\bar{\omega}_{t+1}$ are the same for all entrepreneurs regardless of their net worth.

3.4.2 Derivation of aggregation of across entrepreneurs

Let $f(N_{t+1})$ denote the density of entrepreneurs with net worth N_{t+1}. Then, aggregate average net worth, $\bar{N}_{t+1}$, is

$$\bar{N}_{t+1} = \int_{N_{t+1}} N_{t+1} f(N_{t+1}) dN_{t+1}.$$

We now derive the law of motion of $\bar{N}_{t+1}$. Consider the set of entrepreneurs who in period $t-1$ had net worth N. Their net worth after they have settled with the bank in period t is denoted V_t^N, where

$$V_t^N = R_t^k P_{t-1} P_{k',t-1} \bar{K}_t^N - \Gamma(\bar{\omega}_t; \sigma_{t-1}) R_t^k P_{t-1} P_{k',t-1} \bar{K}_t^N, \tag{3.44}$$

where $\bar{K}_t^N$ is the amount of physical capital that entrepreneurs with net worth N_t acquired in period $t-1$. Clearing in the market for capital requires:

$$\bar{K}_t = \int_{N_t} \bar{K}_t^N f(N_t) dN_t.$$

Multiplying (3.44) by $f(N_t)$ and integrating over all entrepreneurs,

$$V_t = R_t^k P_{t-1} P_{k',t-1} \bar{K}_t - \Gamma(\bar{\omega}_t; \sigma_{t-1}) R_t^k P_{t-1} P_{k',t-1} \bar{K}_t.$$

Writing this out more fully:

$$\begin{aligned}
V_t &= R_t^k P_{t-1} P_{k',t-1} \bar{K}_t - \left\{ [1 - F(\bar{\omega}_t; \sigma_{t-1})] \bar{\omega}_t + \int_0^{\bar{\omega}_t} \omega dF(\omega; \sigma_{t-1}) \right\} R_t^k P_{t-1} P_{k',t-1} \bar{K}_t \\
&= R_t^k P_{t-1} P_{k',t-1} \bar{K}_t \\
&- \left\{ [1 - F(\bar{\omega}_t; \sigma_{t-1})] \bar{\omega}_t + (1-\mu) \int_0^{\bar{\omega}_t} \omega dF(\omega; \sigma_{t-1}) + \mu \int_0^{\bar{\omega}_t} \omega dF(\omega; \sigma_{t-1}) \right\} R_t^k P_{t-1} P_{k',t-1} \bar{K}_t.
\end{aligned}$$

Note that the first two terms in braces correspond to the net revenues of the bank, which must equal $R_{t-1}(P_{t-1} P_{k',t-1} \bar{K}_t - \bar{N}_t)$. Substituting

$$V_t = R_t^k P_{t-1} P_{k',t-1} \bar{K}_t - \left\{ R_{t-1} + \frac{\mu \int_0^{\bar{\omega}_t} \omega dF(\omega; \sigma_{t-1}) R_t^k P_{t-1} P_{k',t-1} \bar{K}_t}{P_{t-1} P_{k',t-1} \bar{K}_t - \bar{N}_t} \right\} (P_{t-1} P_{k',t-1} \bar{K}_t - \bar{N}_t)$$

which implies (2.56) in the main text.

3.4.3 Adjustment to the baseline model when financial frictions are introduced

Consider the households. Households no longer accumulate physical capital, and the FOC (2.42) must be dropped. No other changes need to be made to the household FOCs. Eq. (2.45) can be interpreted as applying to the household's decision to make bank deposits. The household eq-ns (3.33) and (2.43) pertaining to the law of motion and FOC for investment respectively, can be thought of as reflecting that the household builds and sells physical capital, or it can be interpreted as the FOC of many identical competitive firms that build capital (note that each has a state variable in the form of lagged investment). We must add the three equations pertaining to the entrepreneur's loan contract: the law of motion of net worth, the bank's zero profit condition and the optimality condition. Finally, we must adjust the resource constraints to reflect the resources used in bank monitoring and in consumption by entrepreneurs.

We adopt the following scaling of variables, noting that W_t^e is set so that its scaled counterpart is constant

$$n_{t+1} = \frac{\bar{N}_{t+1}}{P_t z_t^+},\ w^e = \frac{W_t^e}{P_t z_t^+}.$$

Dividing both sides of (2.56) by $P_t z_t^+$, we obtain the scaled law of motion for net worth:

$$n_{t+1} = \frac{\gamma_t}{\pi_t \mu_{z^+,t}} [R_t^k p_{k',t-1} \bar{k}_t - R_{t-1}(p_{k',t-1} \bar{k}_t - n_t) - \mu G(\bar{\omega}_t; \sigma_{t-1}) R_t^k p_{k',t-1} \bar{k}_t] + w^e \tag{3.45}$$

for $t = 0, 1, 2, \ldots$. Eq. (3.45) has a simple intuitive interpretation. The first object in square brackets is the average gross return across all entrepreneurs in period t. The two negative terms correspond to what the entrepreneurs pay to the bank, including the interest paid by non-bankrupt entrepreneurs and the resources turned over to the bank by the bankrupt entrepreneurs. Since the bank makes zero profits, the payment to the bank by entrepreneurs must equal bank costs. The term involving R_{t-1} represents the cost of funds loaned to entrepreneurs by the bank, and the term involving μ represents the bank's total expenditures on monitoring costs.

The zero profit condition on banks, (3.43), can be expressed in terms of the scaled variables as

$$\Gamma(\bar{\omega}_{t+1}; \sigma_t) - \mu G(\bar{\omega}_{t+1}; \sigma_t) = \frac{R_t}{R^k_{t+1}} \left(1 - \frac{n_{t+1}}{p_{k',t}\bar{k}_{t+1}}\right) \tag{3.46}$$

for $t = -1, 0, 1, 2, \ldots$. The optimality condition for bank loans is (3.42).

The output equation, (2.49), does not have to be modified. Instead, the resource constraint for domestic homogeneous goods (2.50) needs to be adjusted for the monitoring costs:

$$\begin{aligned} y_t - d_t =& g_t + (1-\omega_c)(p^c_t)^{\eta_c} c_t + (p^i_t)^{\eta_i} \left(i_t + a(u_t)\frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}}\right)(1-\omega_t) \\ &+ \left[\omega_x (p^{m,x}_t)^{1-\eta_x} + (1-\omega_x)\right]^{\frac{\eta_x}{1-\eta_x}} (1-\omega_x)(\mathring{p}^x_t)^{\frac{-\lambda_x}{\lambda_x - 1}} (p^x_t)^{-\eta_f} y^*_t, \end{aligned} \tag{3.47}$$

where

$$d_t = \frac{\mu G(\bar{\omega}_t; \sigma_{t-1}) R^k_t p_{k',t-1}\bar{k}_t}{\pi_t \mu_{z^+,t}}.$$

When the model is brought to the data, measured GDP is y_t adjusted for both monitoring costs and, as in the baseline model, capital utilization costs

$$gdp_t = y_t - d_t - (p^i_t)^{\eta_i}\left(a(u_t)\frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}}\right)(1-\omega_i).$$

Account has to be taken of the consumption by existing entrepreneurs. The net worth of these entrepreneurs is $(1-\gamma_t)V_t$ and it is assumed that a fraction, $1-\Theta$, is taxed and transferred in lump-sum form to households, while the complementary fraction, Θ, is consumed by the existing entrepreneurs. This consumption can be taken into account by subtracting

$$\Theta \frac{1-\gamma_t}{\gamma_t}(n_{t+1} - w^e) z^+_t P_t$$

from the right side of (2.9). In practice we do not make this adjustment because we assume Θ is sufficiently small that the adjustment is negligible.

The financial frictions brings a net increase of two equations (we add (3.42), (3.45) and (3.46), and delete (2.42)) and two variables, n_{t+1} and $\bar{\omega}_{t+1}$. This increases the size of our system to 72 equations in 72 unknowns. The financial frictions also introduce the additional shocks, σ_t and γ_t.

3.5 Measurement equations

Below we report the measurement equations we use to link the model to the data. Our data series for inflation and interest rates are annualized in percentage terms, so we make the same transformation for the model variables, i.e. multiplying by 400:

$$\begin{aligned} R^{data}_t &= 400(R_t - 1) - \vartheta_1 400(R - 1) \\ R^{*,data}_t &= 400(R^*_t - 1) - \vartheta_1 400(R^* - 1) \end{aligned}$$

$$\begin{aligned}
\pi_t^{d,data} &= 400 \log \pi_t - \vartheta_1 400 \log \pi + \varepsilon_{\pi,t}^{me} \\
\pi_t^{c,data} &= 400 \log \pi_t^c - \vartheta_1 400 \log \pi^c + \varepsilon_{\pi^c,t}^{me} \\
\pi_t^{i,data} &= 400 \log \pi_t^i - \vartheta_1 400 \log \pi^i + \varepsilon_{\pi^i,t}^{me} \\
\pi_t^{*,data} &= 400 \log \pi_t^* - \vartheta_1 400 \log \pi^*,
\end{aligned}$$

where $\varepsilon_{i,t}^{me}$ denote the measurement errors for the respective variables. In addition, $\vartheta_1 \in \{0, 1\}$ allows us to handle demeaned and non-demeaned data. In particular, the data for interest rates and foreign inflation are not demeaned. The domestic inflation rates are demeaned.

We use undemeaned first differences in total hours worked,

$$\varDelta \log H_t^{data} = 100 \varDelta \log H_t + \varepsilon_{H,t}^{me}.$$

We use demeaned first-differenced data for the remaining variables. This implies setting $\vartheta_2 = 1$ below:

$$\begin{aligned}
\varDelta \log Y_t^{data} &= 100 \left(\log \mu_{z^+,t} + \varDelta \log \left[y_t - p_t^i a(u_t) \frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}} - d_t - \frac{\kappa}{2} \sum_{j=0}^{N-1} (\tilde{v}_t^j)^2 (1 - \mathcal{F}_t^j) l_t^j \right] \right) \\
&\quad - \vartheta_2 100 (\log \mu_{z^+}) + \varepsilon_{y,t}^{me} \\
\varDelta \log Y_t^{*,data} &= 100 (\log \mu_{z^+,t} + \varDelta \log y_t^*) - \vartheta_2 100 (\log \mu_{z^+}) \\
\varDelta \log C_t^{data} &= 100 (\log \mu_{z^+,t} + \varDelta \log c_t) - \vartheta_2 100 (\log \mu_{z^+}) + \varepsilon_{c,t}^{me} \\
\varDelta \log X_t^{data} &= 100 (\log \mu_{z^+,t} + \varDelta \log x_t) - \vartheta_2 100 (\log \mu_{z^+}) + \varepsilon_{x,t}^{me} \\
\varDelta \log q_t^{data} &= 100 \varDelta \log q_t + \varepsilon_{q,t}^{me} \\
\varDelta \log M_t^{data} &= 100 (\log \mu_{z^+,t} + \varDelta \log Imports_t) - \vartheta_2 100 (\log \mu_{z^+}) + \varepsilon_{M,t}^{me} \\
&= 100 \left[\log \mu_{z^+,t} + \varDelta \log \begin{pmatrix} c_t^m (\mathring{p}_t^{m,c})^{\frac{\lambda_{m,c}}{1-\lambda_{m,c}}} \\ + i_t^m (\mathring{p}_t^{m,i})^{\frac{\lambda_{m,i}}{1-\lambda_{m,i}}} \\ + x_t^m (\mathring{p}_t^{m,x})^{\frac{\lambda_{m,x}}{1-\lambda_{m,x}}} \end{pmatrix} \right] - \vartheta_2 100 (\log \mu_{z^+}) + \varepsilon_{M,t}^{me} \\
\varDelta \log I_t^{data} &= 100 [\log \mu_{z^+,t} + \log \mu_{\psi,t} + \varDelta \log i_t] - \vartheta_2 100 (\log \mu_{z^+} + \log \mu_{\psi}) + \varepsilon_{I,t}^{me} \\
\varDelta \log G_t^{data} &= 100 (\log \mu_{z^+,t} + \varDelta \log g_t) - \vartheta_2 100 (\log \mu_{z^+}) + \varepsilon_{g,t}^{me}
\end{aligned}$$

Note that neither measured GDP nor measured investment include investment goods used for capital maintenance. To calculate measured GDP we also exclude monitoring costs and recruitment costs.

The measurement equation for demeaned first-differenced wages is

$$\varDelta \log (W_t / P_t)^{data} = 100 \varDelta \log \frac{W_t}{z_t^+ P_t} = 100 (\log \mu_{z^+,t} + \varDelta \log \bar{w}_t) - \vartheta_2 100 (\log \mu_{z^+}) + \varepsilon_{W/P,t}^{me}.$$

Finally, we measure demeaned first-differenced net worth and interest rate spread as follows:

$$\begin{aligned}
\varDelta \log N_t^{data} &= 100 (\log \mu_{z^+,t} + \varDelta \log n_t) - \vartheta_2 100 (\log \mu_{z^+}) + \varepsilon_{N,t}^{me} \\
\varDelta \log Spread_t^{data} &= 100 \varDelta \log (z_{t+1} - R_t) = 100 \varDelta \log \left(\frac{\bar{\omega}_{t+1} R_{t+1}^k}{1 - \frac{n_{t+1}}{p_{k',t} \bar{k}_{t+1}}} - R_t \right) + \varepsilon_{Spread,t}^{me}
\end{aligned}$$

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